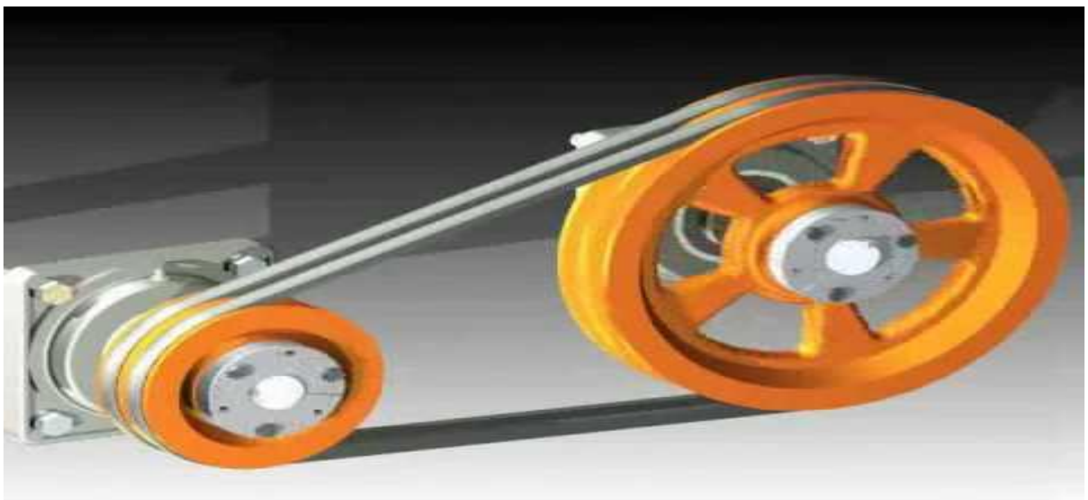
BHADRAK ENGINEERING SCHOOL & TECHNOLOGY
(BEST), ASURALI, BHADRAK

Theory of machine

(Th- 01)

(As per the 2020-21 syllabus of the SCTE&VT, Bhubaneswar, Odisha)



Fourth Semester

Mechanical Engg.

Prepared By: *Er S K Kar*

THEORY OF MACHINES (Th- 01)

Chapter No.	Topics	Periods as per Syllabus	Required period	Expected Marks
01	Simple Mechanism	08	06	15
02	Friction	12	16	25
03	Power Transmission	12	18	25
04	Governor & Flywheel	12	09	15
05	Balancing of Machines	08	04	10
06	Vibration of Machine Parts	08	04	10
TOTAL		**60**	**57**	**100**

INTRODUCTION TO THE SUBJECT

- The subject "Theory of Machines" may be defined as that branch of engineering science which deals with the study of relative motion between various parts of a machine and the forces which act on them.
- A machine is a device which receives energy in some available form and utilizes it to do some particular type of work.

Chapter No.- 01

Simple Mechanism

COURSE CONTENT	
Article No.	**Name of the Article**
1.0	Introduction
1.1	Link
	Kinematic Chain
1.2	Kinematic pair, Lower pair and higher pair
	Mechanism, Machine
1.3	Inversion four bar link mechanism and its inversion
1.4	Cam and followers

CHAPTER 1.0

SIMPLE MECHANISM

Introduction:

We have already discussed that a machine is a device which receives energy in some available form and transforms it into some useful work. A machine consists of a number of parts or bodies. In this chapter, we shall study the mechanisms of the various parts from which the machine is assembled. This is done by making one of the parts as fixed and the relative motion of other parts is determined with respect to the fixed part.

Link/ Kinematic Link/ Element:

Each part of a machine, which moves relative to some the other part, is known as a *kinematic link or link or element*. A link may consist of several parts, which are rigidly fastened together, so that they do not move relative to one another.

Example: Cylinder, piston, connecting rod, etc.

A link should have the following two characteristics:

1. It should have relative motion.
2. It must be a resistant body.

Kinematic Pair:

The two links or elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is completely or successfully constrained (i.e., in a definite direction), the pair is known as kinematic pair.

Example: piston & cylinder; tail stock & lathe bed, etc.

According to the type of contact between the links, the kinematic pair classified into two types:

1. **Lower Pair**: When the two elements of a pair have a surface contact when relative motion takes place and the surface of one element slides over the surface of the other, the pair formed is known as *lower pair*.

   Example: piston & cylinder; lathe spindle & head stock, etc.

2. **Higher Pair**: When the two elements of a pair have a line or point contact when relative motion takes place and the motion between the elements is partly turning and partly sliding, then the pair is known as *higher pair*.
Example: A pair of friction discs; toothed gearing; rope drives, etc.

Kinematic Chain:

When the kinematic pairs are coupled in such a way that the last link is joined to the first link to transmit definite motion (i.e., completely or successfully constrained motion), it is called a *kinematic chain*.

OR

A *kinematic chain* may be defined as a combination of kinematic pairs, joined in such a way that each link forms a part of two pairs and the relative motion between the links or elements is completely or successfully constrained.

Mechanism:

When one of the links of a kinematic chain is fixed, the chain is known as *mechanism*. It may be used for transmitting or transforming motion.

- A mechanism with four links is known as simple mechanism.
- The mechanism with more than four links is known as compound mechanism.

Machine:

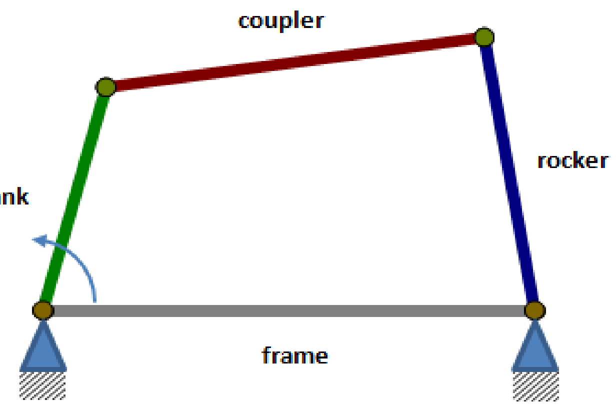
When a mechanism is required to transmit power or to do some particular type of work, it than becomes a *machine*. In such cases, the various links or elements have to be designed to withstand the forces safely.

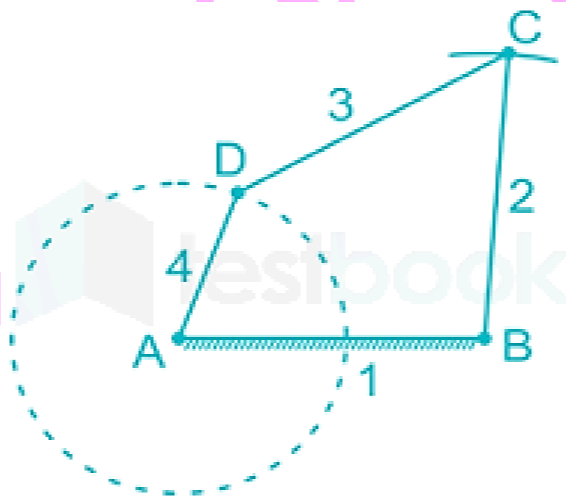
Inversion of Mechanism:

The method of obtaining different mechanisms by fixing different links in a kinematic chain, is known as *inversion of mechanism*.

Four-bar Chain or Quadric Cycle Chain:

The simplest and basic kinematic chain is a four-bar chain or quadric cycle chain. It consists of four links, each of them forms a turning pair at A, B, C and D.





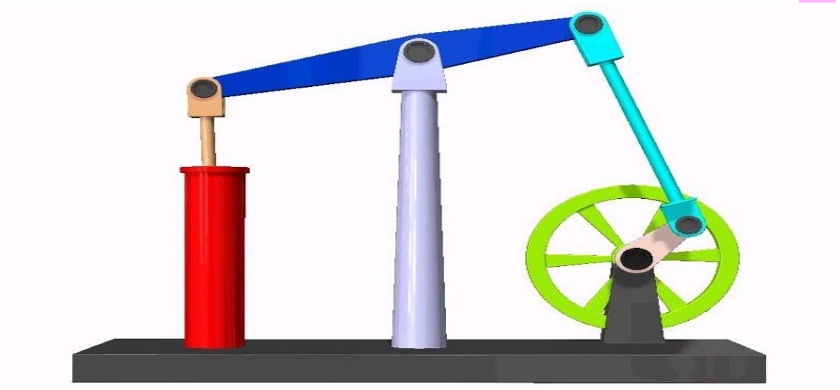
- → In a four-bar chain, one of the links (the shortest link) will make a complete revolution relative to the other three links. Such a link is known as crank or driver. In the figure, link 4 (AD) is a crank.
- → The link 2 (BC), which makes a partial rotation or oscillates is known as lever or rocker or follower.
- → The link 3 (CD), which connects the crank and lever is called connecting rod or coupler.
- → The fixed link 1 (AB), is known as frame of the mechanism.

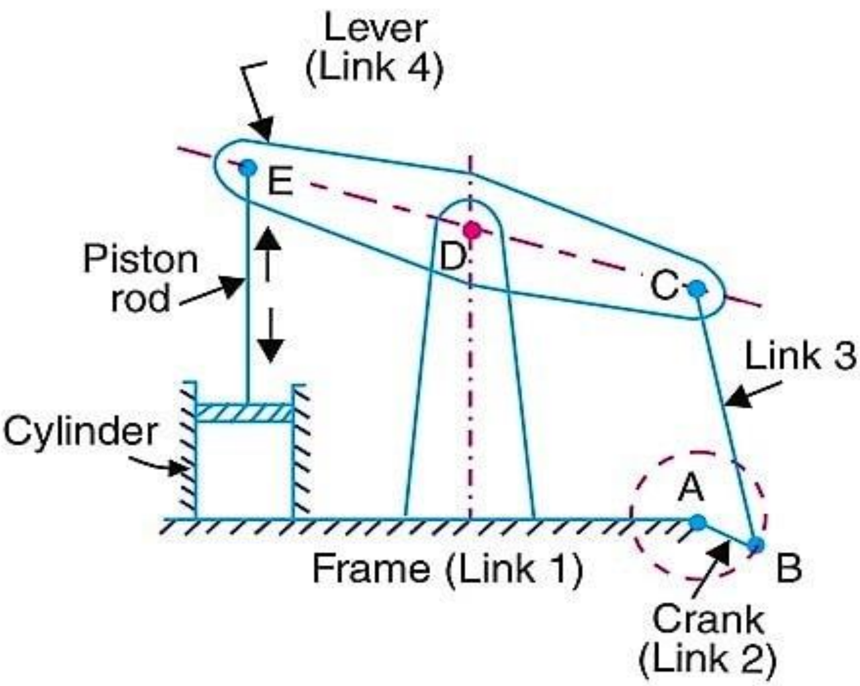
Inversion of four bar chain:

Though there are many inversions of the four-bar chain, yet the following are important from the subject point of view:

1. Beam Engine (Crank & Lever Mechanism):

A part of the mechanism of a Beam engine (also known as crank and lever mechanism) which consists of four links, is shown in the figure.



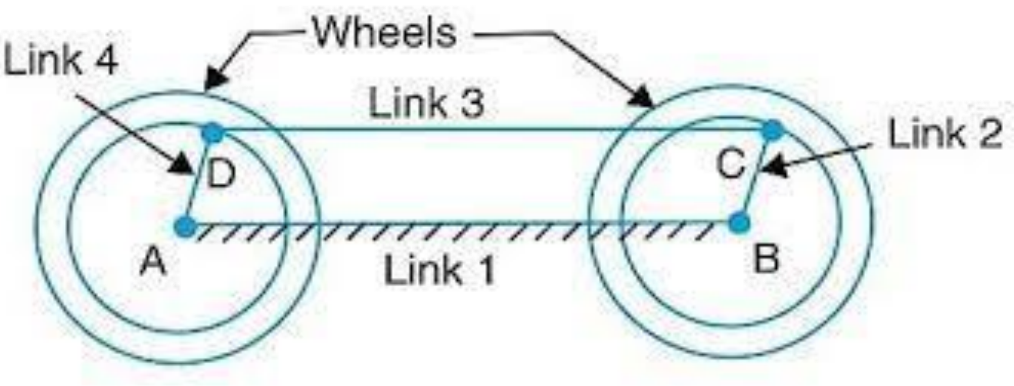


In this mechanism, when the crank rotates about the fixed centre A, the lever oscillates about the fixed centre D. the end E of the lever CDE is connected to a piston rod which reciprocates due to the rotation of the crank.

The purpose of this mechanism is to convert rotary motion into reciprocating motion.

2. Coupling Rod of a Locomotive (Double crank Mechanism):

The mechanism of a coupling rod of a locomotive (also known as double crank mechanism) which consists of four links, is shown in the figure.



In this mechanism, the links AD and BC (having equal lengths) act as cranks and are connected to the respective wheels. The link CD act as coupling rod and the link AB is fixed in order to maintain a constant centre to centre distance between them.

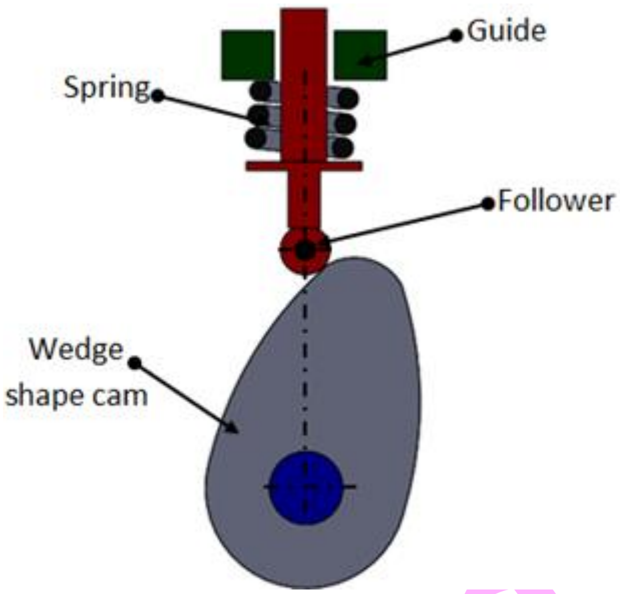
The purpose of this mechanism is to transmit rotary motion from one wheel to the other wheel.

Cam & Followers:

A *cam* is a rotating machine element which gives reciprocating or oscillating motion to another element known as *follower*.

→ The cam and follower have a line contact and hence constitute a higher pair.

→ The cams are usually rotated at uniform speed by a shaft, but the follower motion is pre-determined and will be according to the shape of the cam.



→ The cams are widely used for operating the inlet and exhaust valves of IC Engines, automatic attachment of machineries, etc.

************* END *************

Short Type Questions with Answer

Question No.-01:

Define Theory of Machine.

Ans: The subject "Theory of Machines" may be defined as that branch of engineering science which deals with the study of relative motion between various parts of a machine and the forces which act on them.

Question No.-02:

What is kinematic link?

Ans: Each part of a machine, which moves relative to some the other part, is known as a *kinematic link or link or element*. A link may consist of several parts, which are rigidly fastened together, so that they do not move relative to one another.

Question No.-03:

What are the characteristics of a kinematic link?

Ans: A link should have the following two characteristics:

1. It should have relative motion.
2. It must be a resistant body.

Question No.-04:

Define kinematic Pair.

Ans: The two links or elements of a machine, when in contact with each other, are said to form a pair. If the relative motion between them is completely or successfully constrained (i.e., in a definite direction), the pair is known as kinematic pair.

Question No.-05:

What is lower pair and higher pair?

Ans: When the two elements of a pair have a surface contact when relative motion takes place and the surface of one element slides over the surface of the other, the pair formed is known as *lower pair.*
Example: piston & cylinder; lathe spindle & head stock, etc

When the two elements of a pair have a line or point contact when relative motion takes place and the motion between the elements is partly turning and partly sliding, then the pair is known as *higher pair*.
Example: A pair of friction discs; toothed gearing; rope drives, etc.

Question No.-06:

Define Inversion of Mechanism.

Ans: The method of obtaining different mechanisms by fixing different links in a kinematic chain, is known as *inversion of mechanism.*

Question No.-07:

Define Mechanism.

Ans: When one of the links of a kinematic chain is fixed, the chain is known as *mechanism*. It may be used for transmitting or transforming motion.

Long Type Questions

1. Define four bar chain. Sketch and explain any two inversions of the four-bar chain.
2. Write short notes on:
   (i) Cam & Follower
   (ii) Lower Pair & Higher Pair

CHAPTER NO.- 02

FRICTION

Article No.	Name of the Article
2.1	Revision of topic previously taught
	Screw Friction for square thread, screw jack
	Numerical problem
	Numerical problem
2.2	Bearing and its classification,
	Roller, needle roller & ball bearings.
2.3	Torque transmission in flat pivot bearing
	Torque transmission in conical pivot bearings.
	Numerical problem
2.4	Single flat collar bearing
	Multiple flat collar bearing
	Numerical problem
2.5	Torque transmission for single clutches
	Torque transmission for multiple clutches
	Numerical problem
2.6	Working of simple frictional brakes.
2.7	Working of Absorption type of dynamometer

CHAPTER 2.0

FRICTION

2.0: Revision of Topic previously taught (in Engg Mechanics of 2nd Sem):

1. **Definition of Friction:**

   The opposing force, which acts in the opposite direction of the motion, is called the force of friction or simply friction.

2. **Types of Friction:**

   In general, friction is of the following types:

   a. Static Friction

   b. Dynamic Friction

3. **Limiting Friction:**

   The maximum value of the frictional force, which comes into play, when a body just begins to slide over the surface of the other body, is known as limiting force of friction or simply limiting friction.

4. **Co-efficient of Friction (μ) :**

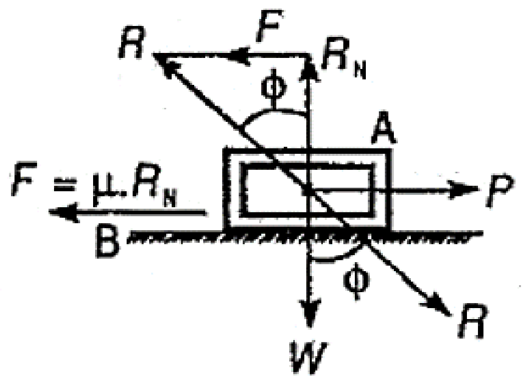
   It is defined as the ratio of the limiting friction (F) to the normal reaction (R_N) between the two bodies. It is denoted by $\boldsymbol{\mu}$.

   Mathematically,

$$\mu = \frac{F}{R_N}$$

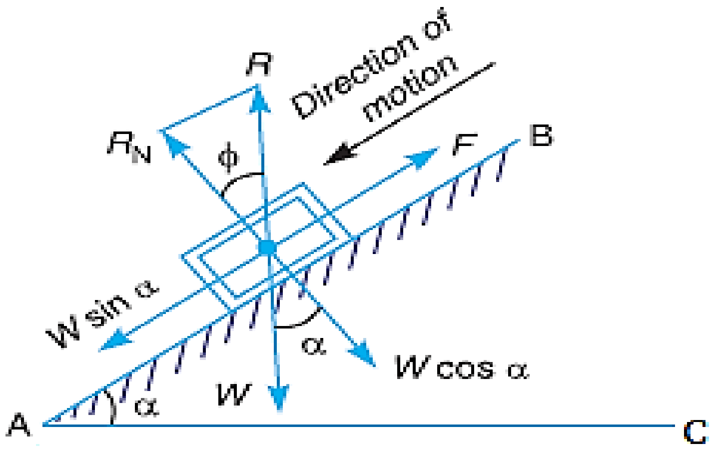
5. **Angle of Friction (φ):**

   The angle made by the resultant of normal reaction and limiting frictional force with the normal reaction is called angle of friction.



6. **Angle of Repose (α):**

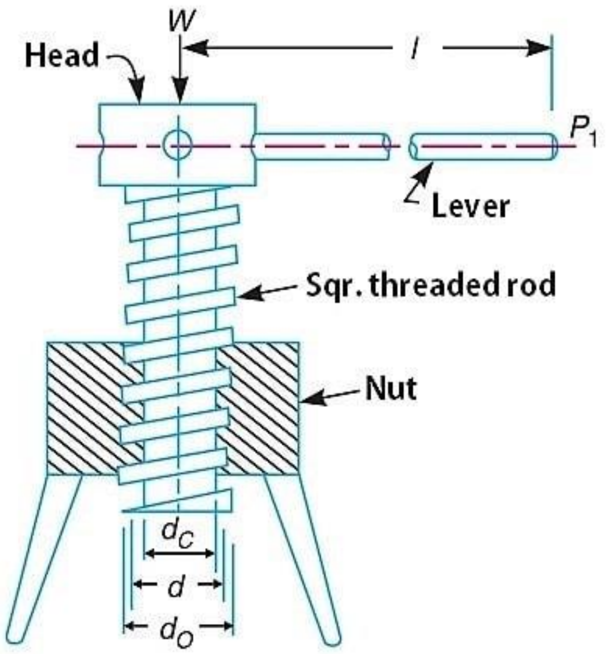
   The minimum angle of the plane at which the body kept on it starts to slide due to its own weight is called angle of repose.

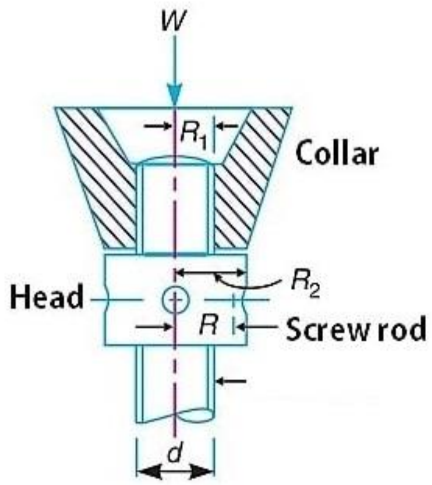


2.1: Friction between nut and screw (for square thread):

The following terms are important for the study of screw friction:

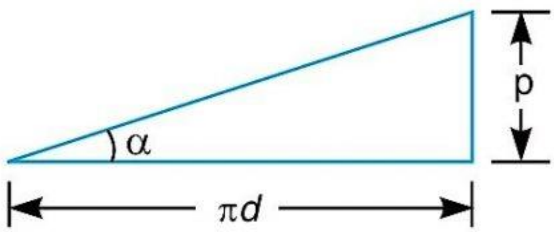
1. **External Thread:** An external thread is a thread on the outside of a member.
2. **Internal Thread:** An internal thread is a thread on the inside of a member.
3. **Pitch:** The distance from a point on a screw thread to a corresponding point on the next thread measured parallel to the axis.
4. **Helix Angle:** It is the slope or inclination of the thread with the horizontal.
   Mathematically,
   $$\tan\alpha = \frac{Lead\ of\ screw}{Circumference\ of\ screw}$$
5. **Lead:** It is the distance, a screw thread advances axially in one turn. On a ***single threaded screw***, the lead and pitch are identical; on ***a double threaded screw,*** the lead is two times the pitch; on ***a triple-headed screw***, the lead is three times the pitch, etc.
6. **Depth of Thread:** The distance between the crest and the base of the thread measured normal to the axis.
7. **Screw Jack:** It is a device, for lifting heavy loads, by applying a comparatively smaller effort at its handle.



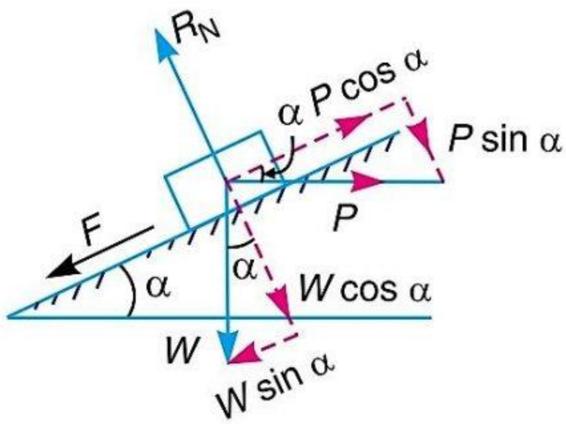


Torque Required to Lift the Load by a Screw Jack:

If one complete turn of a screw thread by imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in below figure:



Development of screw



Forces acting on the screw

Let

p = Pitch of the screw,

d = Mean diameter of the screw,

α = Helix angle,

P = Effort applied at the circumference of the screw to lift the load,

W = Load to be lifted, and

μ = Coefficient of friction, between the screw and nut (= tan ϕ), where ϕ is the friction angle

From the geometry of the figure (left side), we can write

$$\tan\alpha = \frac{p}{\pi d}$$

Since the principle on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the lever of a screw jack may be considered to be horizontal as shown above figure (right side).

Since the load is being lifted, therefore the force of friction ($F = \mu.R_N$) will act downwards. All the forces acting on the screw are shown in the above figure (right side).

Resolving the forces along the plane,

$$P\cos\alpha = W\sin\alpha + F = W\sin\alpha + \mu.R_N \text{ ———(1)}$$

and resolving the forces perpendicular to the plane,

$$R_N = P\sin\alpha + W\cos\alpha \text{ ——(2)}$$

Substituting this value of R_N in equation (1),

$$P\cos\alpha = W\sin\alpha + \mu\,(P\sin\alpha + W\cos\alpha)$$

$$P\cos\alpha = W\sin\alpha + \mu\,P\sin\alpha + \mu\,W\cos\alpha$$

$$P\cos\alpha - \mu\,P\sin\alpha = W\sin\alpha + \mu\,W\cos\alpha$$

$$P\,(\cos\alpha - \mu\sin\alpha) = W\,(\sin\alpha + \mu\cos\alpha)$$

$$P = W \times \frac{\sin\alpha + \mu\cos\alpha}{\cos\alpha - \mu\sin\alpha}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

$$P = W \times \frac{\sin \alpha + \tan \varphi \cos \alpha}{\cos \alpha - \tan \varphi \sin \alpha}$$

Multiplying the numerator and denominator by $\cos \phi$,

$$P = W \times \frac{\sin \alpha \cos \varphi + \sin \varphi \cos \alpha}{\cos \alpha \cos \varphi - \sin \alpha \sin \varphi}$$

$$= W \times \frac{\sin(\alpha + \varphi)}{\cos(\alpha + \varphi)} = W \tan(\alpha + \varphi)$$

$$\therefore \boldsymbol{P = W \tan(\alpha + \varphi)}$$

Torque required to overcome friction between the screw and nut

$$T_1 = P \times (d/2)$$

$$T_1 = W \tan(\alpha + \phi) \times (d/2)$$

When the axial load is taken up by a thrust collar or a flat surface, as shown in above figure, so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$T_2 = \mu_1 \times W \times (R_1 + R_2)/2$$

$$T_2 = \mu_1 \times W \times R$$

Where,

- R_1 and R_2 = Outside and inside radii of the collar
- R = Mean radius of the collar, and
- μ_1 = Coefficient of friction for the collar

$\therefore$ Total torque required to overcome friction (i.e. to rotate the screw),

$$T = T_1 + T_2$$

$T = (P \times (d/2)) + (\mu_1 \times W \times R)$

If an effort P1 is applied at the end of a lever of arm length *l*, then the total torque required to overcome friction must be equal to the torque applied at the end of the lever, i.e.

$T = P \times (d/2) = P_1 \times l$

Numerical No.-01:

An electric motor-driven power screw moves a nut in a horizontal plane against a force of 75 kN at a speed of 300 mm/min. The screw has a single square thread of 6 mm pitch on a major diameter of 40 mm. The coefficient of friction at the screw threads is 0.1. Estimate power of the motor.

Solution:

Given Data: W= 75 kN = 75 X 10^3 N ; v = 300 mm/min ; p = 6 mm ; d_0 = 40 mm

μ = tan ϕ = 0.1.

We know that,

mean diameter of screw, d = d_0 – p/2 = 40 – 6/2 = 37 mm = 0.037 m

and, tan α = $\frac{p}{\pi d} = \frac{6}{\pi \times 37} = 0.0516$

∴ Force required at the circumference of the screw,

$$P = W \tan(\alpha + \varphi) = W \times \frac{\tan\alpha + \tan\varphi}{1 - \tan\alpha \times \tan\varphi} = 75 \text{ X } 10^3 \times \frac{0.0516+0.1}{1-0.0516 \times 0.1} =$$

11.43 X 10^3 N

And, torque required to overcome friction,

T = P x d/2 = 11.43 X 10^3 x 0.037/2 = 211.45 Nm

We know that speed of the screw,

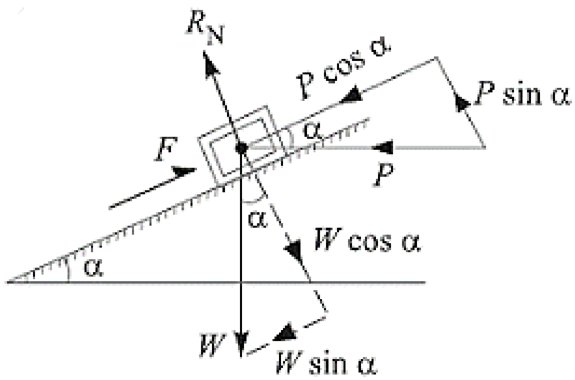
N = $\frac{Speed\ of\ the\ nut}{Pitch\ of\ the\ screw} = \frac{300}{6} = 50$ rpm

And, Angular Speed, $\omega = \frac{2\pi N}{60} = \frac{2\pi \times 50}{60} = 5.24$ rad/s

∴ Power of the motor = T. ω = 211.45 x 5.24 = 1108 W = 1.108 kW **(Ans)**

Torque Required to Lower the Load by a Screw Jack:

If one complete turn of a screw thread by imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in below figure:



Let

p = Pitch of the screw,

d = Mean diameter of the screw,

α = Helix angle,

P = Effort applied at the circumference of the screw to lower the load,

W = Load to be lowered, and

μ = Coefficient of friction, between the screw and nut (= tan ϕ), where ϕ is the friction angle

From the geometry of the figure (left side), we can write

$$\tan\alpha = \frac{p}{\pi d}$$

Since the principle on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the lever of a screw jack may be considered to be horizontal as shown above figure.

Since the load is being lowered, therefore the force of friction ($F = \mu.R_N$) will act upwards. All the forces acting on the screw are shown in the above figure.

Resolving the forces along the plane,

$$P \cos \alpha = F - W \sin \alpha = \mu . R_N - W \sin \alpha \quad ---(1)$$

and resolving the forces perpendicular to the plane,

$$R_N = W \cos \alpha - P \sin \alpha \quad --(2)$$

Substituting this value of R_N in equation (1),

$$P \cos \alpha = \mu (W \cos \alpha - P \sin \alpha) - W \sin \alpha$$

$$P \cos \alpha = \mu W \cos \alpha - \mu P \sin \alpha - W \sin \alpha$$

$$P \cos \alpha + \mu P \sin \alpha = \mu W \cos \alpha - W \sin \alpha$$

$$P (\cos \alpha + \mu \sin \alpha) = W (\mu \cos \alpha - \sin \alpha)$$

$$P = W \times \frac{\mu \cos \alpha - \sin \alpha}{\cos \alpha + \mu \sin \alpha}$$

Substituting the value of $\mu = \tan \phi$ in the above equation, we get

$$P = W \times \frac{\tan \varphi \cos \alpha - \sin \alpha}{\cos \alpha + \tan \varphi \sin \alpha}$$

Multiplying the numerator and denominator by $\cos \phi$,

$$P = W \times \frac{\sin \varphi \cos \alpha + \sin \alpha \cos \varphi}{\cos \alpha \cos \varphi + \sin \alpha \sin \varphi}$$

$$= W \times \frac{\sin (\varphi - \alpha)}{\cos (\varphi - \alpha)} = W \tan(\varphi - \alpha)$$

$$\therefore \boldsymbol{P = W \tan(\varphi - \alpha)}$$

Torque required to overcome friction between the screw and nut

$$T = P \times (d/2)$$

$$\mathbf{T = W \tan(\phi - \alpha) \times (d/2)}$$

Numerical No.-02:

The mean diameter of a square threaded screw jack is 50 mm. The pitch of the thread is 10 mm. The coefficient of friction is 0.15. What force must be applied at the end of a 0.7 m long lever, which is perpendicular to the longitudinal axis of the screw to raise a load of 20 kN and to lower it?

Solution:

Given Data: W= 20 kN = 20 X 10^3 N ; l = 0.7 m ; p = 10 mm ; d = 50 mm = 0.05 m ;

$\mu = \tan \phi = 0.15$.

We know that, $\tan \alpha = \frac{p}{\pi d} = \frac{10}{\pi \times 50} = 0.0637$

Let P_1 = Force required at the end of the lever

Force required to raise the load:

We know that, force required at the circumference of the screw

$$= W \tan(\alpha + \varphi) = W \times \frac{\tan \alpha + \tan \varphi}{1 - \tan \alpha \times \tan \varphi} = 20 \text{ X } 10^3 \times \frac{0.0637 + 0.15}{1 - 0.0637 \times 0.15} = 4314 \text{ N}$$

Now, the force required at the end of the lever may be found out by the relation,

$$P_1 . l = P . d/2$$

$$\text{or } P_1 = \frac{Pd}{2l} = \frac{4314 \times 0.05}{2 \times 0.7} = 154 \text{ N } \textbf{(Ans)}$$

Force required to lower the load:

We know that, force required at the circumference of the screw

$$= W \tan(\varphi - \alpha) = W \times \frac{\tan \varphi - \tan \alpha}{1 + \tan \varphi \times \tan \alpha} = 20 \text{ X } 10^3 \times \frac{0.15 - 0.0637}{1 - 0.0637 \times 0.15} = 1710 \text{ N}$$

Now, the force required at the end of the lever may be found out by the relation,

$$P_1 . l = P . d/2$$

$$\text{or } P_1 = \frac{Pd}{2l} = \frac{1710 \times 0.05}{2 \times 0.7} = 61 \text{ N}$$ **(Ans)**

Efficiency of a Screw Jack:

It may be defined as the ratio between the ideal effort (i.e., the effort required to move the load, neglecting friction) to the actual effort (i.e., the effort required to move the load, considering friction).

Mathematically,

$$\textbf{Efficiency, } \eta = \frac{\text{Ideal Effort}}{\text{Actual Effort}} = \frac{\tan\alpha}{\tan(\alpha+\varphi)}$$

Maximum Efficiency of a Screw Jack:

$$\eta_{max} = \frac{1 - \sin\varphi}{1 + \sin\varphi}$$

2.2: Bearing & Its Classification:

→ A **bearing** is a machine element that constrains relative motion to only the desired motion, and reduces friction between moving parts.

→ The main purpose of bearings is to prevent direct metal to metal contact between two elements that are in relative motion. This prevents friction, heat generation and ultimately, the wear and tear of parts. It also reduces energy consumption as sliding motion is replaced with low friction rolling.

→ They also transmit the load of the rotating element to the housing. This load may be radial, axial, or a combination of both.

Classification of Bearing:

1. Ball Bearings:

Ball bearings are one of the most common types of bearing classes used. It consists of a row of balls as rolling elements. They are trapped between two annulus shaped metal pieces. These metal pieces are known as races. The inner race is free to rotate while the outer race is stationary.

2. **Roller Bearings:**

Roller bearings contain cylindrical rolling elements instead of balls as load carrying elements between the races. An element is considered a roller if its length is longer than its diameter (even if only slightly). Since they are in line contact with the inner and outer races (instead of point contact as in the case of ball bearings), they can support greater loading.

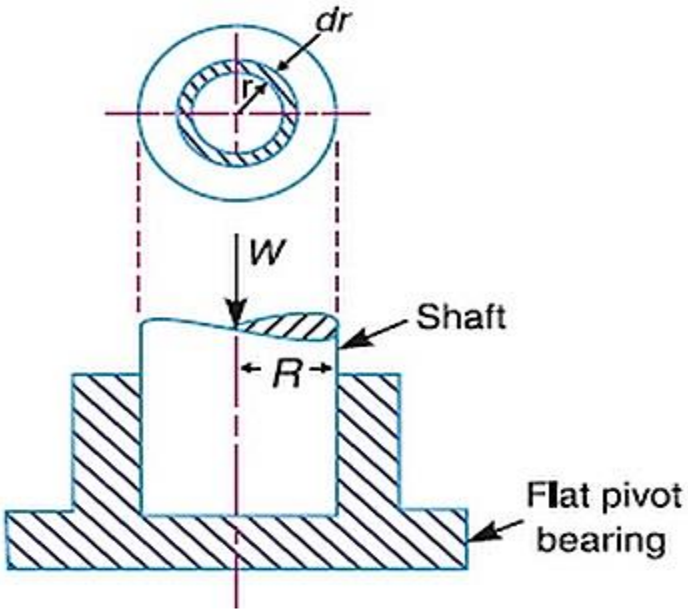
3. **Needle Roller Bearing:**

Needle roller bearing is a special type of roller bearings that has cylindrical rollers that resemble needles because of their small diameter. Normally, the length of rollers in roller bearings is only slightly more than its diameter. When it comes to needle bearing, the length of rollers exceeds their diameter by at least four times.

2.3: Torque transmission in Flat Pivot Bearing/ Foot step Bearing:

The rotating shafts are frequently subjected to axial thrust. The bearing surfaces such as pivot and collar bearings are used to take this axial thrust.

The bearing surfaces placed at the end of the shaft to take the axial thrust are known as pivots. The pivots may have flat surface or conical surface.



Let

$W =$ *Load transmitted over the bearing surface*

$R =$ *Radius of bearing surface*

$p =$ *Intensity of pressure per unit area of bearing surface*

$\mu =$ *Co-efficient of friction*

We will consider the following two cases:

1. When there is a uniform pressure
2. When there is a uniform wear

Considering Uniform Pressure:

When the pressure is uniformly distributed over the bearing area, then

$$p = \frac{W}{\pi r^2}$$

Considering a ring of radius r and thickness dr of the bearing area.

Then, area of bearing surface, $A = 2\pi r. dr$

Load transmitted to the ring, $\delta W = p. A = p. 2\pi r. dr$ --------- (i)

Frictional Resistance, $F_r = \mu. \delta W = \mu. p. 2\pi r. dr$

Frictional torque on the ring, $T_r = F_r. r = \mu. p. 2\pi r. dr\ . r = 2\pi\mu p r^2 dr$ ----(ii)

Integrating the above equation within the limits from 0 to R for the total frictional torque on the pivot bearing.

Total frictional torque, $T = \int_0^R 2\pi\mu p r^2 dr = 2\pi\mu p \int_0^R r^2 dr$

$$= 2\pi\mu p \left[\frac{r^3}{3}\right]_0^R = 2\pi\mu p \frac{R^3}{3} = \frac{2}{3}\pi\mu p\, R^3$$

$$= \frac{2}{3}\pi\mu. \frac{W}{\pi R^2} R^3 \quad \text{-----}\left(Since, p = \frac{W}{\pi R^2}\right)$$

$$\therefore \boldsymbol{T = \frac{2}{3}\mu W R}$$

Considering Uniform Wear:

For Uniform wear, $p. r = C$ (a constant)

Or, $p = \frac{C}{r}$

Load transmitted to the ring, $\delta W = p. A = p. 2\pi r. dr$ (From Eq^n-i)

$$= \frac{C}{r} \times 2\pi r. dr = 2\pi C. dr$$

Total load transmitted to the bearing,

$$W = \int_0^R 2\pi C. dr = 2\pi C \int_0^R dr$$

$$= 2\pi C\,[r]_0^R = 2\pi C R$$

Or, $C = \frac{W}{2\pi R}$

Frictional torque on the ring, $T_r = 2\pi\mu p r^2 dr = 2\pi\mu \times \frac{C}{r} \times r^2 dr = 2\pi\mu. C. r dr$

Total Frictional Torque, $T = \int_0^R 2\pi\mu. C. r dr = 2\pi\mu C \int_0^R r. dr$

$$= 2\pi\mu C \left[\frac{r^2}{2}\right]_0^R = 2\pi\mu C \frac{R^2}{2} = \frac{2}{2} \pi\mu C R^2$$

$$= \pi\mu . \frac{W}{2\pi R} R^2 \quad \text{-----}\left(Since, C = \frac{W}{2\pi R}\right)$$

$$\therefore \boldsymbol{T = \frac{1}{2} \mu W R}$$

Numerical No.-03:

A vertical shaft 150 mm in diameter rotating at 100 rpm rests on a flat end footstep bearing. The shaft carries a vertical load of 20 kN. Assuming uniform pressure distribution and co-efficient of friction equals to 0.05, estimate power lost in friction.

Solution:

Given Data: D = 150 mm or R = 75 mm = 0.075 m ; W= 20 kN = 20 X 10^3 N ; N = 100 rpm or $\omega = \frac{2\pi\times100}{60}$ = 10.47 rad/s ; μ = 0.05.

We know that for uniform pressure distribution,

Total frictional torque, $T = \frac{2}{3} \mu W R = \frac{2}{3} \times 0.05 \times 20 \times 10^3 \times 0.075 = 50\ Nm$

Power lost in friction, $P = T\omega = 50 \times 10.47 = 523.5$ W **(Ans)**

Torque transmission in Conical Pivot Bearing:

Let

$W =$ *Load transmitted over the bearing surface*

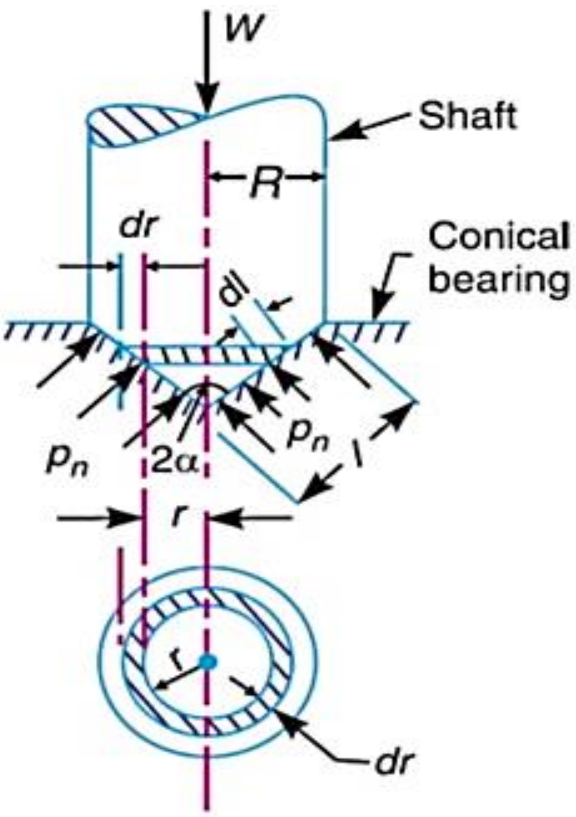
$R =$ *Radius of bearing surface*

$p_n =$ *Intensity of pressure normal to the cone*

$\mu =$ *Co-efficient of friction*

Consider a small ring of radius r and thickness dr. Let dl is the length of ring along the cone, such that $dl = dr. cosec\ \alpha$

Area of the ring, $A = 2\pi r.dl = 2\pi r.dr.cosec\ \alpha$



Considering Uniform Pressure:

We know that normal load acting on the ring, δW_n = Normal pressure x Area

$= p_n \times 2\pi r.dr.cosec\ \alpha$

And Vertical Load acting on the ring, $\delta W = \text{Vertical component of } \delta W_n = \delta W_n \sin\alpha$

$$= p_n \times 2\pi r.dr.cosec\ \alpha.\sin\alpha = p_n \times 2\pi r.dr$$

Total vertical load transmitted to the bearing,

$$W = \int_0^R p_n \times 2\pi r.dr = 2\pi p_n \int_0^R r.dr$$

$$= 2\pi p_n \left[\frac{r^2}{2}\right]_0^R = 2\pi p_n \frac{R^2}{2} = \pi R^2 p_n$$

$$\text{Or, } p_n = \frac{W}{\pi R^2}$$

Frictional Resistance, $F_r = \mu.\delta W_n = \mu.p_n \times 2\pi r.dr.cosec\ \alpha = 2\pi\mu p_n cosec\ \alpha.r.dr$

Frictional torque on the ring,

$$T_r = F_r.r = 2\pi\mu p_n cosec\ \alpha.r.dr.r = 2\pi\mu p_n cosec\ \alpha.r^2.dr$$

Integrating the above equation within the limits from 0 to R for the total frictional torque on the conical pivot bearing.

Total frictional torque, $T = \int_0^R 2\pi\mu p_n cosec\ \alpha.r^2.dr = 2\pi\mu p_n cosec\ \alpha \int_0^R r^2 dr$

$$= 2\pi\mu p_n cosec\ \alpha \left[\frac{r^3}{3}\right]_0^R = 2\pi\mu p_n cosec\ \alpha\ \frac{R^3}{3} = \frac{2}{3}\pi R^3 \mu p_n cosec\ \alpha$$

Substituting the value of p_n in the above equation

$$T = \frac{2}{3}\pi R^3 \times \mu \times \frac{W}{\pi R^2} \times cosec\ \alpha$$

$$\therefore \boldsymbol{T = \frac{2}{3}\mu WR.cosec\ \alpha}$$

Considering Uniform Wear:

For Uniform wear, $p_r.r = C$ (a constant)

Or, $p_r = \frac{C}{r}$

Load transmitted to the ring, $\delta W = p_r.2\pi r.dr$ (From Eq^n-i)

$$= \frac{C}{r} \times 2\pi r.dr = 2\pi C.dr$$

Total load transmitted to the bearing,

$$W = \int_0^R 2\pi C.dr = 2\pi C \int_0^R dr$$

$$= 2\pi C\ [r]_0^R = 2\pi CR$$

Or, $C = \frac{W}{2\pi R}$

Frictional torque on the ring,

$$T_r = 2\pi\mu p_r cosec\ \alpha . r^2 . dr = 2\pi\mu \times \frac{C}{r} \times cosec\ \alpha . r^2 dr =$$
$$2\pi\mu . C . cosec\ \alpha . rdr$$

Total Frictional Torque, $T = \int_0^R 2\pi\mu . C . cosec\ \alpha . rdr =$
$2\pi\mu C . cosec\ \alpha\ \int_0^R r . dr$

$$= 2\pi\mu C\ cosec\ \alpha \left[\frac{r^2}{2}\right]_0^R = 2\pi\mu C\ cosec\ \alpha \frac{R^2}{2} = \frac{2}{2}\ \pi\mu C\ cosec\ \alpha . R^2$$

$$= \pi\mu . \frac{W}{2\pi R}\ cosec\ \alpha\ R^2 \quad \text{-----}\left(Since, C = \frac{W}{2\pi R}\right)$$

$$\therefore \boldsymbol{T = \frac{1}{2}\ \mu W R . cosec\ \alpha}$$

Numerical No.-04:

A conical pivot supports a vertical shaft of 200 mm diameter. It is subjected to a load of 30 kN. The angle of the cone is 120⁰ and the co-efficient of friction is 0.025. find the power lost in friction when the speed is 140 rpm, assuming 1. Uniform pressure and 2. Uniform wear.

Solution:

Given Data: D = 200 mm or R = 100 mm = 0.1 m ; W= 30 kN = 30 X 10^3 N ; N = 140 rpm or $\omega = \frac{2\pi \times 140}{60}$ = 14.66 rad/s ; μ = 0.025 ; 2α = 120^0 or α = 60^0

1. Power lost in friction assuming uniform pressure:

We know that total frictional torque,

$$T = \frac{2}{3}\ \mu W R . cosec\ \alpha = \frac{2}{3} \times 0.025 \times 30 \times 10^3 \times 0.1 \times cosec\ 60^0 = 57.7 \text{ Nm}$$

Power lost in friction, $P = T\omega = 57.7 \times 14.66 = 846$ W **(Ans)**

2. Power lost in friction assuming uniform wear:

We know that total frictional torque,

$$T = \frac{1}{2}\ \mu W R . cosec\ \alpha = \frac{1}{2} \times 0.025 \times 30 \times 10^3 \times 0.1 \times cosec\ 60^0 = 43.3 \text{ Nm}$$

Power lost in friction, $P = T\omega = 43.3 \times 14.66 = 634.8$ W **(Ans)**

CHAPTER NO.- 02

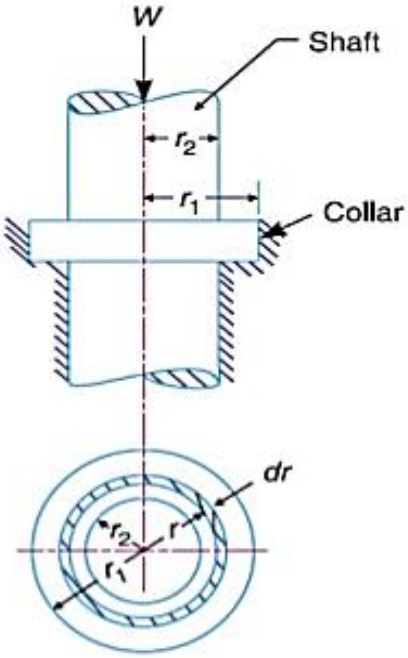
FRICTION (Part- II)

Article No.	Name of the Article
2.4	Single flat collar bearing
	Multiple flat collar bearing
	Numerical problem
2.5	Torque transmission for single clutches
	Torque transmission for multiple clutches
	Numerical problem
2.6	Working of simple frictional brakes.
2.7	Working of Absorption type of dynamometer
	Probable Questions

2.4: Flat Collar Bearing/ Thrust Bearing:

Collar bearings are used to take the axial thrust of the rotating shafts. There may be a single collar or multiple collar bearings.

Single Flat Collar Bearing:



Let

r_1 = External radius of the collar

r_2 = Internal radius of the collar

Area of the bearing surface, $A = \pi[r_1^2 - r_2^2]$

Considering uniform pressure:

When the pressure is uniformly distributed over the bearing surface, then the intensity of pressure,

$$p = \frac{W}{A} = \frac{W}{\pi[r_1^2 - r_2^2]}$$

We know that, Frictional torque on the ring, $T_r = 2\pi\mu p r^2 dr$

Integrating the above equation within the limits from r_2 to r_1 for the total frictional torque on the collar bearing.

Total frictional torque, $T = \int_{r_2}^{r_1} 2\pi\mu p r^2 dr = 2\pi\mu p \int_{r_2}^{r_1} r^2 dr$

$$= 2\pi\mu p \left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi\mu p \left(\frac{{r_1}^3 - {r_2}^3}{3}\right)$$

$$= 2\pi\mu \times \frac{W}{\pi[r_1^2 - r_2^2]} \times \left(\frac{{r_1}^3 - {r_2}^3}{3}\right) \quad \text{-----}\left(Since, p = \frac{W}{\pi[r_1^2 - r_2^2]}\right)$$

$$\therefore \boldsymbol{T = \frac{2}{3}\mu W\left(\frac{{r_1}^3 - {r_2}^3}{r_1^2 - r_2^2}\right)}$$

Considering Uniform Wear:

Load transmitted to the ring, $\delta W = p_r . 2\pi r . dr = \frac{C}{r} \times 2\pi r . dr = 2\pi C . dr$

Total load transmitted to the bearing,

$$W = \int_{r_2}^{r_1} 2\pi C . dr = 2\pi C \int_{r_2}^{r_1} dr$$

$$= 2\pi C \, [r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

Or, $C = \frac{W}{2\pi(r_1 - r_2)}$

Frictional torque on the ring, $T_r = 2\pi\mu . C . rdr$

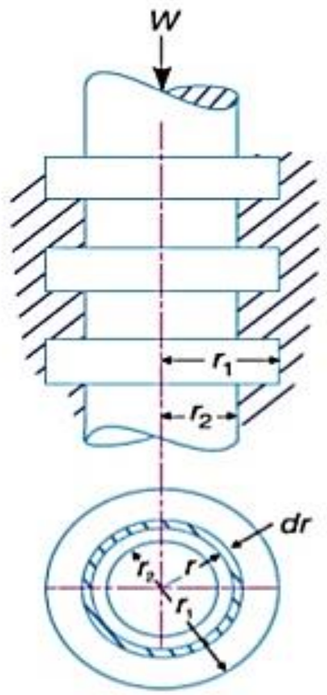
Total Frictional Torque, $T = \int_{r_2}^{r_1} 2\pi\mu . C . rdr = 2\pi\mu C \int_{r_2}^{r_1} r . dr$

$$= 2\pi\mu C \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu C \left(\frac{{r_1}^2 - {r_2}^2}{2}\right) = \frac{2}{2}\pi\mu C \left[{r_1}^2 - {r_2}^2\right]$$

$$= \pi\mu . \frac{W}{2\pi(r_1 - r_2)} \left[{r_1}^2 - {r_2}^2\right] \quad \text{-----}\left(Since, C = \frac{W}{2\pi(r_1 - r_2)}\right)$$

$$\therefore \boldsymbol{T = \frac{1}{2}\mu W(r_1 - r_2)}$$

Multiple Flat Collar Bearing:



Total torque transmitted in a multi collared shaft remains

$$T = \frac{2}{3}\,\mu W\left(\frac{{r_1}^3 - {r_2}^3}{r_1^2 - r_2^2}\right) \quad \textbf{for uniform pressure}$$

$$= \frac{1}{2}\,\mu W(r_1 + r_2) \quad \textbf{for uniform wear}$$

Numerical No.-05:

A thrust shaft of a ship has 6 collars of 600 mm external diameter and 300 mm internal diameter. The total thrust from the propeller is 100 kN. If the co-efficient of friction is 0.12 and speed of the engine 90 rpm, find the power absorbed in friction at the thrust block, assuming 1. Uniform pressure and 2. Uniform wear.

Solution:

Given Data: n = 6 ; d_1 = 600 mm or r_1 = 300 mm ; d_2 = 300 mm or r_2 = 150 mm; W= 100 kN = 100 X 10^3 N ; N = 90 rpm or $\omega = \frac{2\pi \times 90}{60}$ = 9.426 rad/s ; μ = 0.12

1. Power absorbed in friction assuming uniform pressure:

We know that total frictional torque transmitted,

$$T = \frac{2}{3}\mu W\left(\frac{{r_1}^3 - {r_2}^3}{r_1^2 - r_2^2}\right) = \frac{2}{3} \times 0.12 \times 100 \times 10^3\left(\frac{300^3 - 150^3}{300^2 - 150^2}\right)$$

$$= 2800 \times 10^3\ N.mm = 2800\ Nm$$

Power absorbed in friction,

$$P = T\omega = 2800 \times 9.426 = 26\,400 \text{ W} = 26.4 \text{ kW}$$ **(Ans)**

2. Power absorbed in friction assuming uniform wear:

We know that total frictional torque transmitted,

$$T = \frac{1}{2}\mu W(r_1 + r_2) = \frac{1}{2} \times 0.12 \times 100 \times 10^3(300 + 150)$$

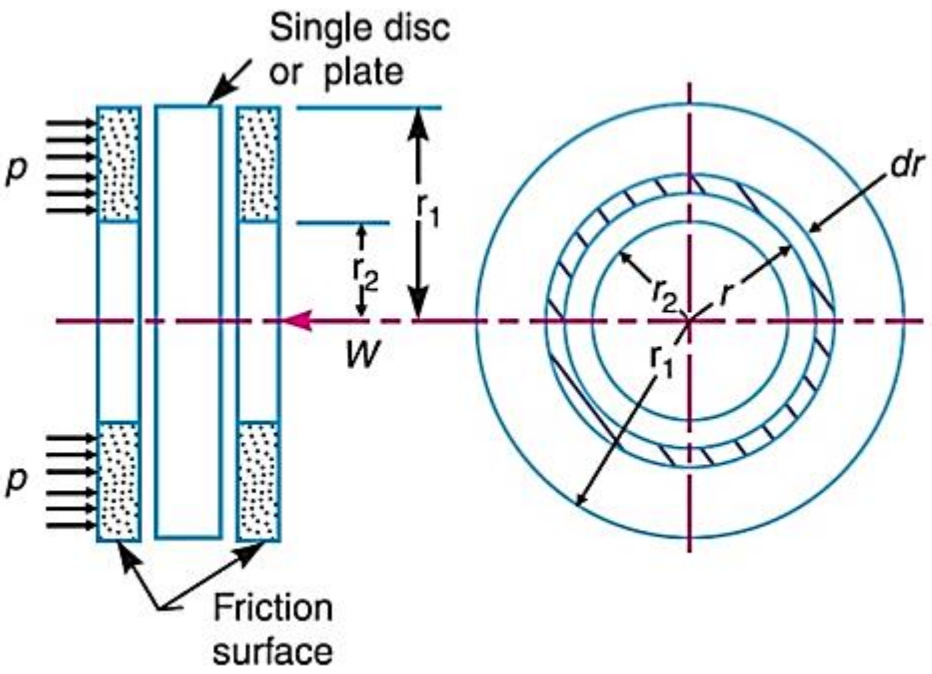
$$= 2700 \times 10^3\ N.mm = 2700\ Nm$$ **(Ans)**

Power absorbed in friction,

$$P = T\omega = 2700 \times 9.426 = 25\,450 \text{ W} = 25.45 \text{ kW}$$ **(Ans)**

2.5: Torque transmission in Single Disc or Plate Clutch:

Consider two friction surfaces, maintained in contact by an axial thrust W, as shown in the figure.



Let

$T = Torque\ transmitted\ by\ the\ clutch$

$p = Intensity\ of\ axial\ pressure$

$r_1 = External\ radius\ of\ the\ collar$

$r_2 = Internal\ radius\ of\ the\ collar$

$\mu = Co\text{-}efficient\ of\ friction$

Considering a ring of radius r and thickness dr of the bearing area.

Then, area of bearing surface, $A = 2\pi r.dr$

Load transmitted to the ring, $\delta W = p.A = p.2\pi r.dr$

Frictional Resistance, $F_r = \mu.\delta W = \mu.p.2\pi r.dr$

Frictional torque on the ring, $T_r = F_r.r = \mu.p.2\pi r.dr\ .r = 2\pi\mu p r^2 dr$

We will consider the following two cases:

1. When there is a uniform pressure
2. When there is a uniform wear

Considering Uniform Pressure:

When the pressure is uniformly distributed over the bearing area, then

$$p = \frac{W}{A} = \frac{W}{\pi[r_1^2 - r_2^2]}$$

Integrating the equation of frictional torque acting on the ring, within the limits from r_2 to r_1 for the total frictional torque.

Total frictional torque, $T = \int_{r_2}^{r_1} 2\pi\mu p r^2 dr = 2\pi\mu p \int_{r_2}^{r_1} r^2 dr$

$$= 2\pi\mu p \left[\frac{r^3}{3}\right]_{r_2}^{r_1} = 2\pi\mu p \left(\frac{{r_1}^3 - {r_2}^3}{3}\right)$$

$$= 2\pi\mu \times \frac{W}{\pi[r_1^2 - r_2^2]} \times \left(\frac{{r_1}^3 - {r_2}^3}{3}\right) \quad \text{-----}\left(Since, p = \frac{W}{\pi[r_1^2 - r_2^2]}\right)$$

$$\therefore \boldsymbol{T = \frac{2}{3}\mu W\left(\frac{{r_1}^3 - {r_2}^3}{r_1^2 - r_2^2}\right) = \mu W R}$$

Where, $R =$ Mean radius of the friction surface

$$= \frac{2}{3}\left(\frac{{r_1}^3 - {r_2}^3}{r_1^2 - r_2^2}\right)$$

Considering Uniform Wear:

Load transmitted to the ring, $\delta W = p_r . 2\pi r . dr = \frac{C}{r} \times 2\pi r . dr = 2\pi C . dr$

Total load transmitted to the bearing,

$$W = \int_{r_2}^{r_1} 2\pi C . dr = 2\pi C \int_{r_2}^{r_1} dr$$

$$= 2\pi C \, [r]_{r_2}^{r_1} = 2\pi C(r_1 - r_2)$$

Or, $C = \frac{W}{2\pi(r_1 - r_2)}$

Frictional torque on the ring, $T_r = 2\pi\mu . C . rdr$

Total Frictional Torque, $T = \int_{r_2}^{r_1} 2\pi\mu . C . rdr = 2\pi\mu C \int_{r_2}^{r_1} r . dr$

$$= 2\pi\mu C \left[\frac{r^2}{2}\right]_{r_2}^{r_1} = 2\pi\mu C \left(\frac{{r_1}^2 - {r_2}^2}{2}\right) = \frac{2}{2} \pi\mu C \left[{r_1}^2 - {r_2}^2\right]$$

$$= \pi\mu . \frac{W}{2\pi(r_1 - r_2)} \left[{r_1}^2 - {r_2}^2\right] \quad \text{-----}\left(Since, C = \frac{W}{2\pi(r_1 - r_2)}\right)$$

$$\therefore \boldsymbol{T = \frac{1}{2} \mu W(r_1 + r_2) = \mu W R}$$

Where, $R =$ Mean radius of the friction surface

$$= \frac{(\boldsymbol{r_1} + \boldsymbol{r_2})}{2}$$

Little Further:

1. In general, total frictional torque acting on the friction surface (or on the clutch) is given by: $T = n\mu W R$

   Where, n = number of pairs of friction or contact surfaces

R = Mean radius of friction surface

$$= \frac{2}{3}\left(\frac{{r_1}^3 - {r_2}^3}{r_1^2 - r_2^2}\right) \qquad \text{for uniform pressure}$$

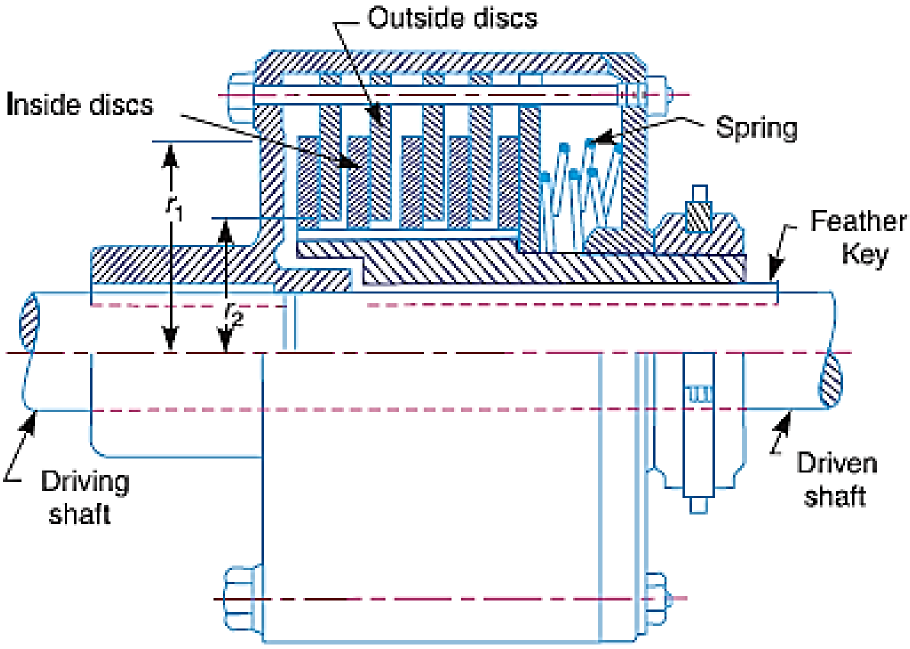
$$= \frac{(r_1 + r_2)}{2} \qquad \text{for uniform wear}$$

2. For a single disc or plate clutch, n = 2.
3. Since the intensity of pressure is maximum at the inner radius (r_2), therefore, $p_{max} \cdot r_2 = C$.
4. Since the intensity of pressure is minimum at the outer radius (r_1), therefore, $p_{min} \cdot r_1 = C$.
5. The average pressure on the friction or contact surface is given by:

$$p_{av} = \frac{W}{\pi[r_1^2 - r_2^2]}.$$

6. In case of a new clutch, the intensity of pressure is approximately uniform but in an old clutch the uniform wear theory is more approximate.
7. The uniform pressure theory gives a higher frictional torque than the uniform wear theory. Therefore, in case of friction clutches, uniform wear should be considered, unless otherwise stated.

Multiple Disc or Plate Clutch:



- A multiple disc clutch may be used when a large torque is to be transmitted.
- The inside discs (usually of steel) are fastened to the driven shaft to permit axial motion (except for the last disc).
- The outside discs (usually of bronze) are held by bolts and are fastened to the housing which is keyed to the driving shaft.
- The multiple disc clutches are extensively used in motor cars, machine tools, etc.

Let

n_1 = No. of discs on the driving shaft

n_2 = No. of discs on the driven shaft

then, No. of pairs of contact surfaces, $n = n_1 + n_2 - 1$

Total Frictional Torque acting on the friction surfaces,

$$\boldsymbol{T = n\mu W R}$$

Where,

R = Mean radius of friction surface

$$= \frac{2}{3}\left(\frac{{r_1}^3 - {r_2}^3}{r_1^2 - r_2^2}\right) \qquad \text{for uniform pressure}$$

$$= \frac{(r_1 + r_2)}{2} \qquad \text{for uniform wear}$$

Numerical No.-06:

A single plate clutch with both sides effective, has outer and inner diameters 300 mm and 200 mm respectively. The maximum intensity of pressure at any point in the contact surface is not to exceed 0.1 N/mm². If the co-efficient of friction is 0.3, determine the power transmitted by a clutch at a speed of 2500 rpm.

Solution:

Given Data: n = 2 ; d_1 = 300 mm or r_1 = 150 mm ; d_2 = 200 mm or r_2 = 100 mm;

p_{max} = 0.1 N/mm² ; N = 2500 rpm or $\omega = \frac{2\pi \times 2500}{60}$ = 261.8 rad/s ; μ = 0.3

Since the intensity of pressure is maximum at inner radius, therefore for uniform wear, $p_{max} \cdot r_2 = C$ or C = 0.1 X 100 = 10 N/mm

The axial thrust, $W = 2\pi C(r_1 - r_2) = 2\pi \times 10(150 - 100) =$ 3142 N

Mean Radius of friction surfaces, $R = \frac{(r_1 + r_2)}{2} = \frac{(150+100)}{2}$ = 125 mm = 0.125 m

Torque transmitted,
$T = n\mu WR = 2 \times 0.3 \times 3142 \times 0.125$ = 235.65 Nm

Power Transmitted by clutch,

$$P = T\omega = 235.65 \times 261.8 = 61\,693 \text{ W} = 61.693 \text{ kW}$$ **(Ans)**

Numerical No.-07:

A multiple plate clutch has five plates having four pairs of active friction surfaces. If the intensity of pressure at any point in the contact surface is not to exceed 0.127 N/mm², determine the power transmitted at a speed of 500 rpm. The outer and inner radii of friction surfaces are 125 mm and 75 mm resp. Assume uniform wear and the co-efficient of friction = 0.3.

Solution:

Given Data: n = 4 ; r_1 = 125 mm ; r_2 = 75 mm ; p_{max} = 0.127 N/mm^2 ;

N = 500 rpm or $\omega = \frac{2\pi \times 500}{60}$ = 52.4 rad/s ; μ = 0.3

Since the intensity of pressure is maximum at inner radius, therefore for uniform wear, $p_{max} \cdot r_2 = C$ or C = 0.127 X 75 = 9.525 N/mm

The axial thrust, $W = 2\pi C(r_1 - r_2) = 2\pi \times 9.525\ (125 - 75) = 2990$ N

Mean Radius of friction surfaces, $R = \frac{(r_1 + r_2)}{2} = \frac{(125+75)}{2}$ = 100 mm = 0.1 m

Torque transmitted,
$T = n\mu WR = 2 \times 0.3 \times 2990 \times 0.1$ = 358.8 Nm

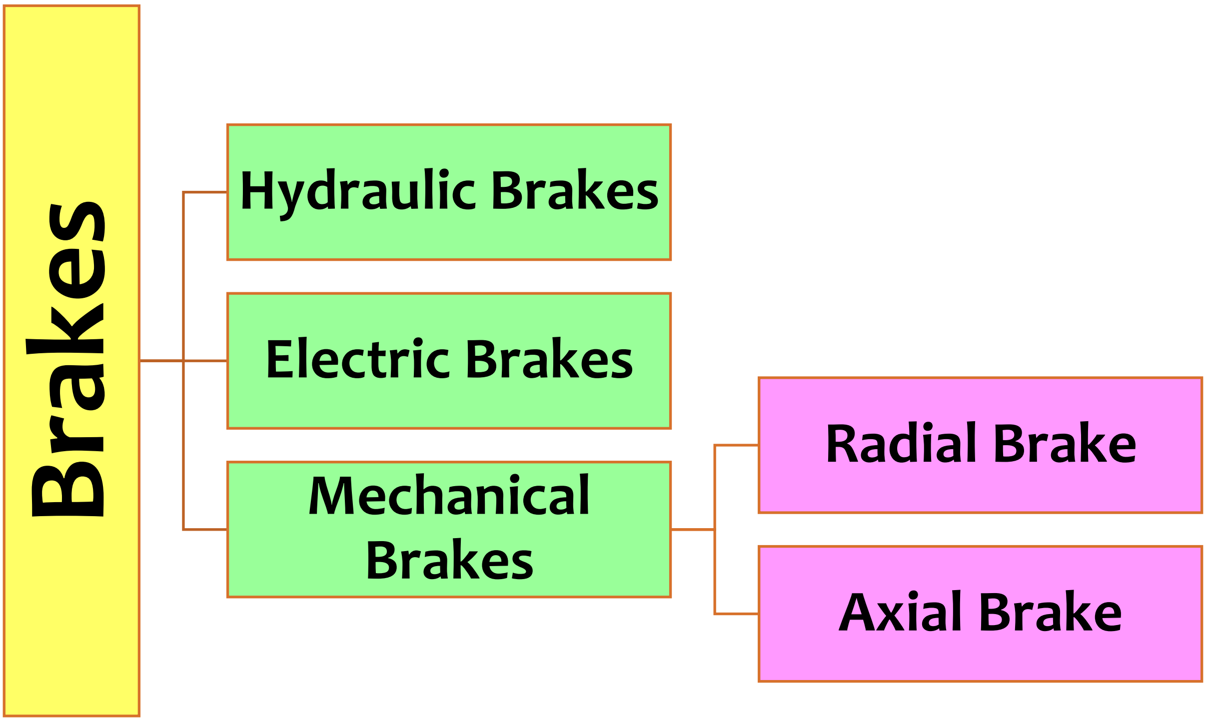
Power Transmitted by clutch,

$$P = T\omega = 358.8 \times 52.4 = 18\,800 \text{ W} = 18.8 \text{ kW}$$ **(Ans)**

2.6: Working of Simple Frictional Brake:

- A brake is a device by means of which artificial frictional resistance is applied to a moving machine member, in order to retard or stop the motion of a machine.
- In the process of performing this function, the absorbs either kinetic energy of the moving member or potential energy given up by objects being lowered by hoists, elevators, etc.

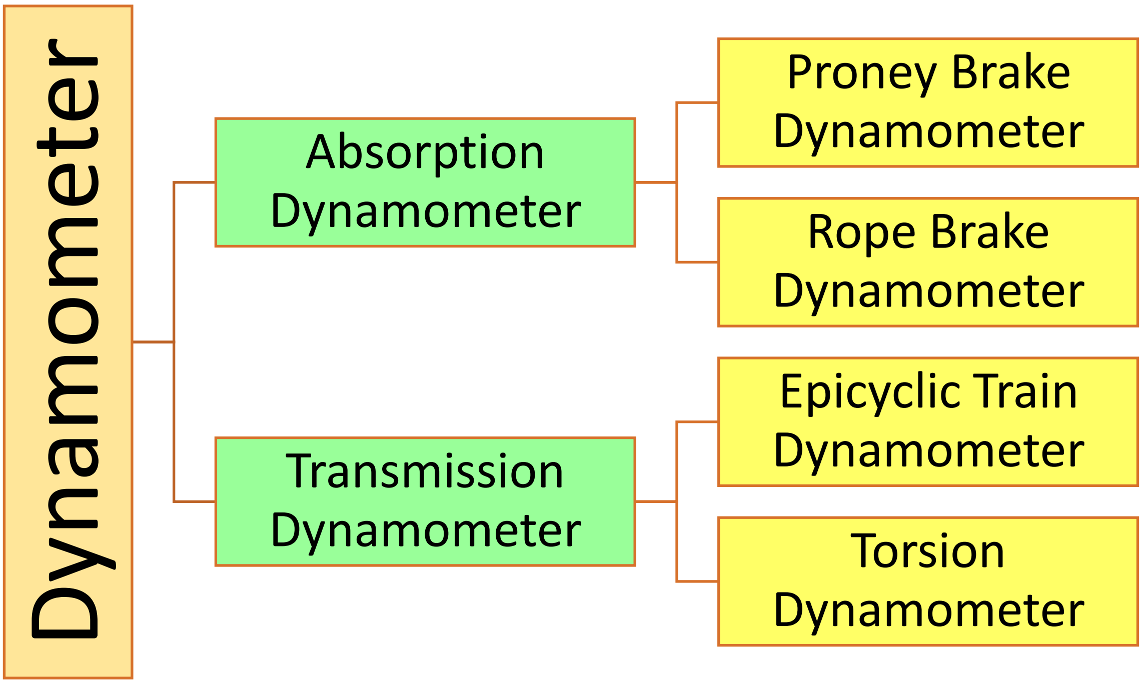
- The energy absorbed by brakes is dissipated in the form of heat.
- The major functional difference between a clutch and a brake is that a clutch is used to keep the driving and driven member moving together, whereas brakes are used to stop a moving member or to control its speed.



2.7: Dynamometer:

Dynamometer is a brake but in addition it has a device to measure the frictional resistance. Knowing the frictional resistance, we may obtain the torque transmitted and hence the power of the engine.

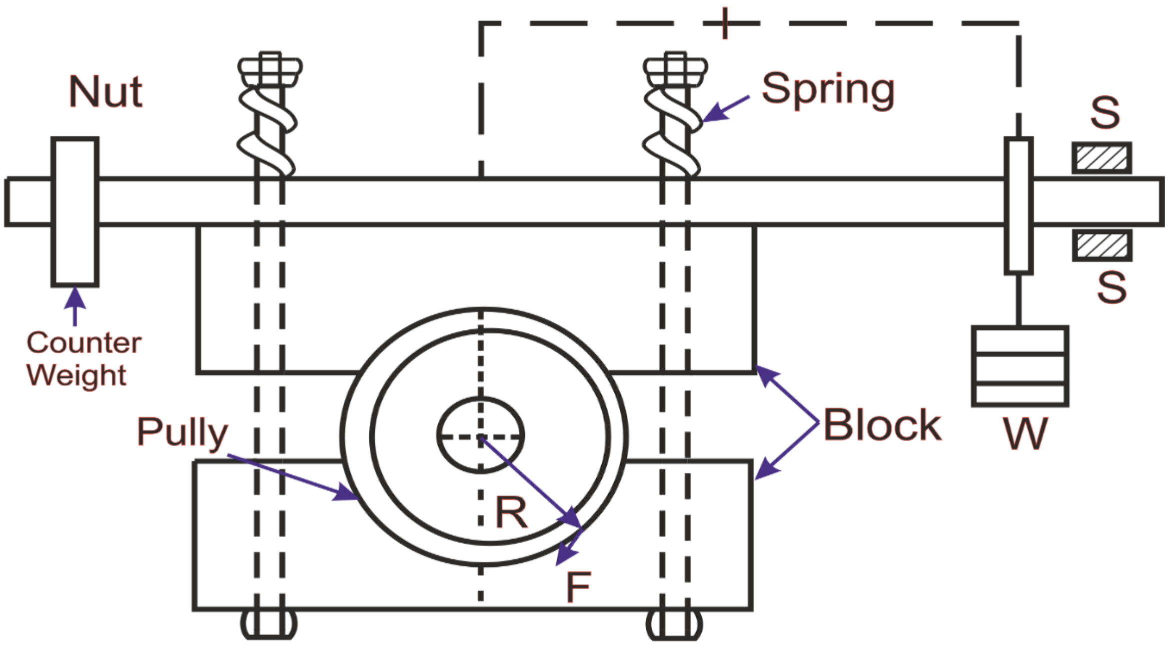
Dynamometers are classified as follows:

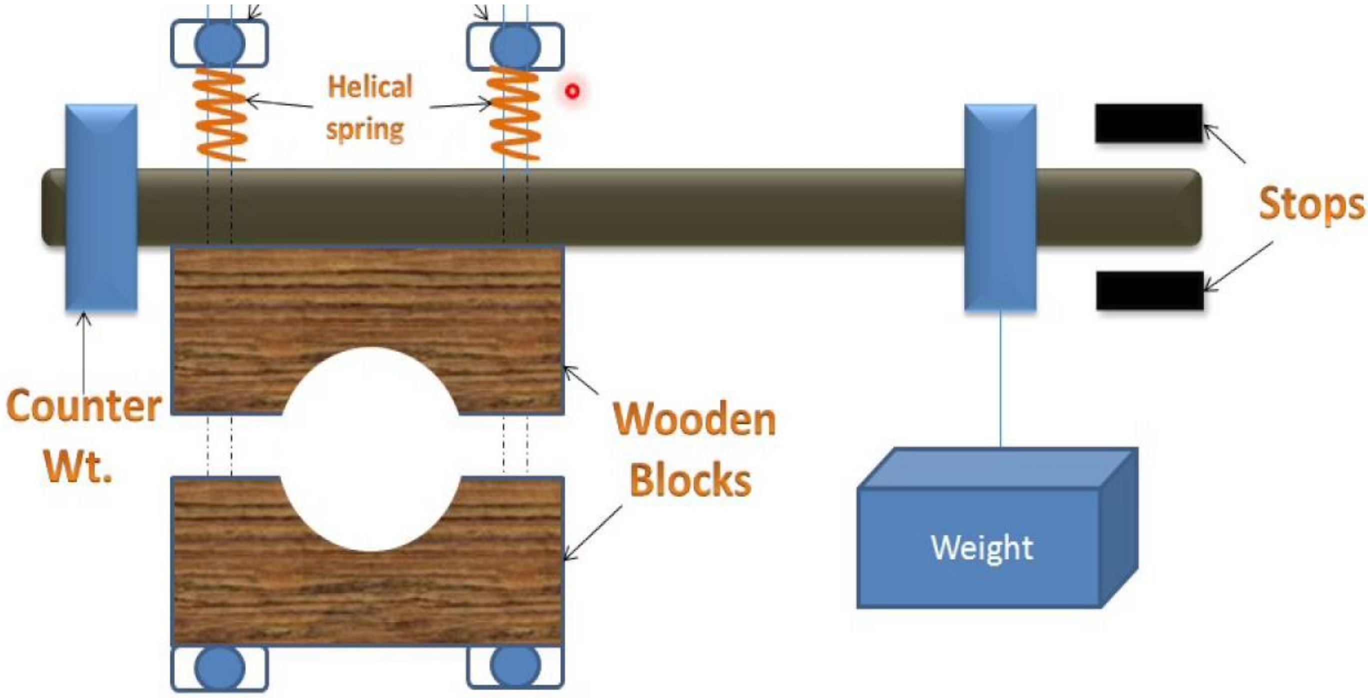


1. **Absorption Dynamometer:**
   In the absorption dynamometers, the entire energy or power produced by the engine is absorbed by the friction resistances of the brake and it is transformed into heat, during the process of measurement.
2. **Transmission Dynamometer:** In the transmission dynamometers, the energy is not wasted in friction but is used for doing work. The energy or power produced by the engine is transmitted through dynamometer to some other machines, where power developed is suitably measured.

Prony brake dynamometer:

A simplest form of absorption type dynamometer is a Prony brake dynamometer. It consists of two wooden blocks placed around a pully fixed to the shaft of an engine whose power is required to be measured. The blocks are clamped by means of two nuts and bolts. A helical spring is provided between the nut and the upper block to adjust the pressure on the pully to control its speed. The upper block has a long lever attached to it and carries a weight 'W' at its outer end. A counter weight is placed at the other end of the lever which balances the brake when unloaded. Two stops S, S are provided to limit the motion of the lever.





When the brake is to be put in operation, the long end of the lever is loaded with suitable weights W and the nuts are tightened until the engine shaft runs at a constant speed and the lever is in horizontal position. Under these conditions, the moment due to the weight 'W' must balance the moment of the frictional resistance between the blocks and the pully.

Let,

W = Weight at the outer end of the lever in newtons.

l =Horizontal distance of the weight W from the centre of the pully in meters.

F = Fractional resistance between the blocks and the pully in newtons,

R = Radius of the pully in meters, and

N = Speed of the shaft in r.p.m.

We know that the moment of the frictional resistance or torque on the shaft,

$$T = W.l = F.R \text{ Nm}$$

Work done in one revolution, $T = \text{Torque } \times \text{Angle turned in radians} =$
$T.2\pi \text{ Nm}$

Work done per minute $= \ T.2\pi.N \text{ Nm}$

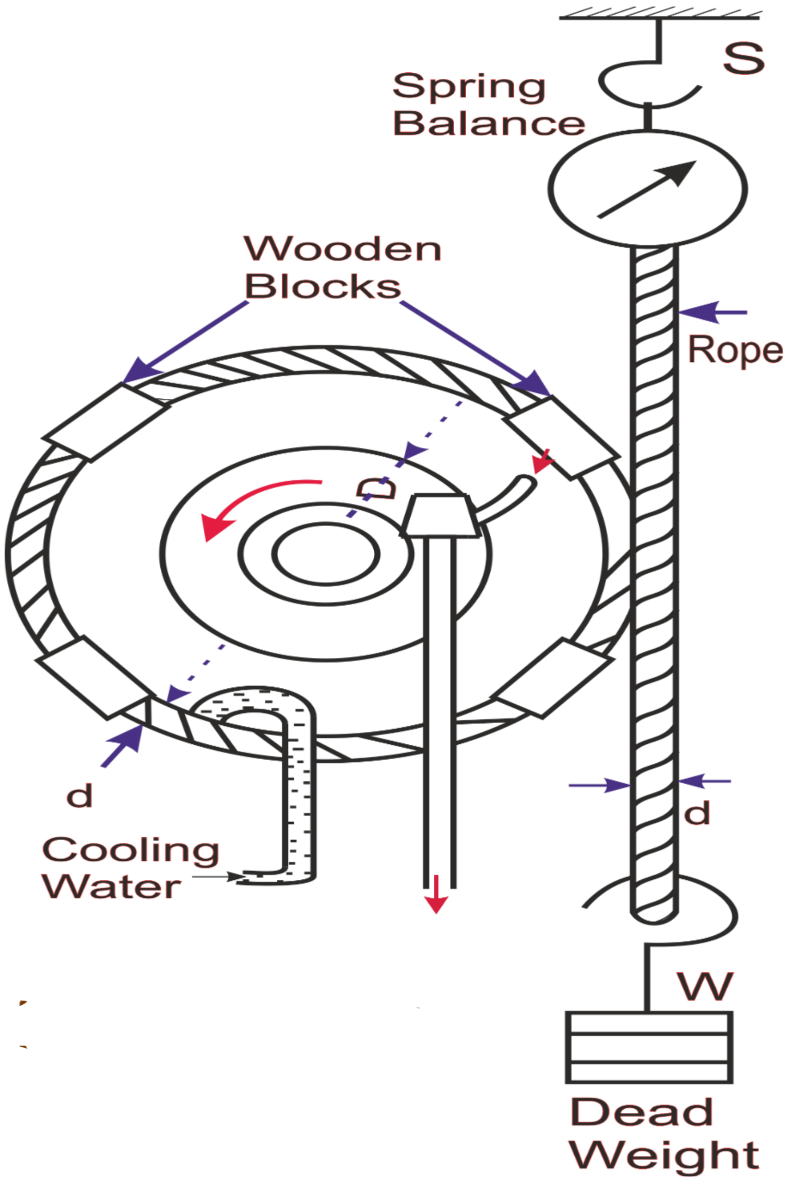
We know that brake power of the engine,

$$BP = \frac{\text{work done per min}}{60} = \frac{T.2\pi.N}{60} = \frac{W.l.2\pi.N}{60} \text{ Watts}$$

Rope brake dynamometer:

It is another form of absorption type dynamometer which is most commonly used to measure the brake power of an engine. It consists one, two or more ropes wound around the flywheel or rim of a pully fixed rigidly to the shaft of an engine. The upper end of the ropes is attached to a spring balance, while the lower end of the ropes is kept in position by applying a dead weight as shown in the figure. In order to prevent the slipping of the rope over the flywheel, wooden blocks are placed at intervals around the circumference of the flywheel.

In the operation of the brake, the engine is made to run at a constant speed. The frictional torque, due to the rope, must be equal to the torque being transmitted by the engine.



Let,

W = Dead load in newtons,

S = Spring balance reading in newtons,

D = Diameter of the wheel in meters,

d = Diameter of rope in meters, and

N = Speed of the engine shaft in r.p.m.

Net load on the brake $= (W - S)\ \mathrm{N}$

We know that distance moved in one revolution $= \pi(D + d)\ \mathrm{m}$

Work done per revolution $= (W - S)\ \pi\ (D + d)\ \mathrm{Nm}$

and work done per minute $= (W - S)\ \pi\ (D + d)\ N\ \mathrm{Nm}$

So, break power of the engine is,

$$BP = \frac{\text{work done per min}}{60} = \frac{(W - S)\ \pi\ (D + d)\ N}{60}\ \text{Watts}$$

If the diameter of the rope (d) is neglected, then brake power of the engine,

$$BP = \frac{(W - S)\ \pi\ D\ N}{60}\ \text{Watts}$$

****

Short Type Questions with Answer

Question No.-01:

Define limiting friction.

Ans: The maximum value of the frictional force, which comes into play, when a body just begins to slide over the surface of the other body, is known as limiting force of friction or simply limiting friction.

Question No.-02:

Define co-efficient of friction.

Ans: It is defined as the ratio of the limiting friction (F) to the normal reaction (R_N) between the two bodies. It is denoted by $\boldsymbol{\mu}$.

Mathematically,

$$\mu = \frac{F}{R_N}$$

Question No.-03:

Define angle of friction.

Ans: The angle made by the resultant of normal reaction and limiting frictional force with the normal reaction is called angle of friction.

Question No.-04:

Define angle of repose.

Ans: The minimum angle of the plane at which the body kept on it starts to slide due to its own weight is called angle of repose.

Question No.-05:

Define pitch of a thread.

Ans: The distance from a point on a screw thread to a corresponding point on the next thread measured parallel to the axis.

Question No.-06:

What is efficiency of a screw jack?

Ans: It may be defined as the ratio between the ideal effort (i.e., the effort required to move the load, neglecting friction) to the actual effort (i.e., the effort required to move the load, considering friction).

Question No.-07:

State the formula for maximum efficiency of a screw jack.

Ans: The formula for maximum efficiency of a screw jack is

$$\eta_{max} = \frac{1 - \sin\varphi}{1 + \sin\varphi}$$

Question No.-08:

Differentiate between clutch and brake.

Ans: The major functional difference between a clutch and a brake is that a clutch is used to keep the driving and driven member moving together, whereas brakes are used to stop a moving member or to control its speed.

Question No.-09:

Define dynamometer.

Ans: Dynamometer is a brake but in addition it has a device to measure the frictional resistance. Knowing the frictional resistance, we may obtain the torque transmitted and hence the power of the engine.

Question No.-10:

Define absorption dynamometer.

Ans: In the absorption dynamometers, the entire energy or power produced by the engine is absorbed by the friction resistances of the brake and it is transformed into heat, during the process of measurement.

Question No.-11:

Define transmission dynamometer.

Ans: In the transmission dynamometers, the energy is not wasted in friction but is used for doing work. The energy or power produced by the engine is transmitted through dynamometer to some other machines, where power developed is suitably measured.

Long Type Questions

1. Derive an expression for the torque transmitted by a Flat pivot bearing considering uniform pressure theory.
2. Derive an expression for the torque transmitted by a Conical pivot bearing considering uniform wear theory.
3. Derive an expression for the torque transmitted by a single collar bearing considering uniform pressure theory.
4. Derive an expression for the torque transmitted by a single plate clutch.
5. Write short notes on ball bearing, roller bearing and needle roller bearing.
6. Describe with neat sketch, the working of an absorption type dynamometer.
7. Write short notes on simple frictional brake.
8. All numerical.

CHAPTER NO.- 03 (Part- I)

POWER TRANSMISSION

Article No.	Name of the Article
3.1	Concept of power transmission
3.2	Type of drives, belt, gear and chain drive.
3.3	Computation of velocity ratio
	Length of belt of open belt drive with and without slip
	Length of belt of cross belt drive with and without slip
3.4	Ratio of belt tensions,
	Centrifugal tension
	Initial Tension.
3.5	Power transmitted by the belt. Numerical problem
	Numerical problem
3.6	Determine The belt thickness and width for permissible stress for open and cross belt drive considering centrifugal tension
3.7	V-belts and V-belts pulleys.
3.8	Concept of crowning of pulleys.
3.9	Gear drives and its terminology.
3.10	Gear trains, Working principle of simple gear trains.
	Working principle of compound gear trains.
	Working principle of reverted gear trains.
	Working principle of epicyclic gear trains.
	Numerical problem
	Numerical problem

CHAPTER 3.0

POWER TRANSMISSION

3.1: Concept of Power Transmission (Mechanical):

Power transmission is the movement of energy from its place of generation to a location where it is applied to perform useful work.

Mechanical power transmission is the transfer of energy from where it's generated to a place where it is used to perform work using simple machines, linkages and mechanical power transmission elements.

Mechanical power transmission and its elements are used for the following reasons.

1. Generated power or energy can be converted into a useful form.
2. Physical constraints limit the power generation at the place where its used hence it can be transferred from source to a place where it is needed.
3. It can be used to change direction and magnitude such as speed or torque.
4. It can be used to change the type of energy i.e., rotational to linear and vice versa.

There are different types of power transmission elements, such as:

- Shafts & Couplings
- Power screws
- Gears & Gear trains
- Brakes & Clutches
- Belts, Ropes & Pulleys
- Chains & sprockets

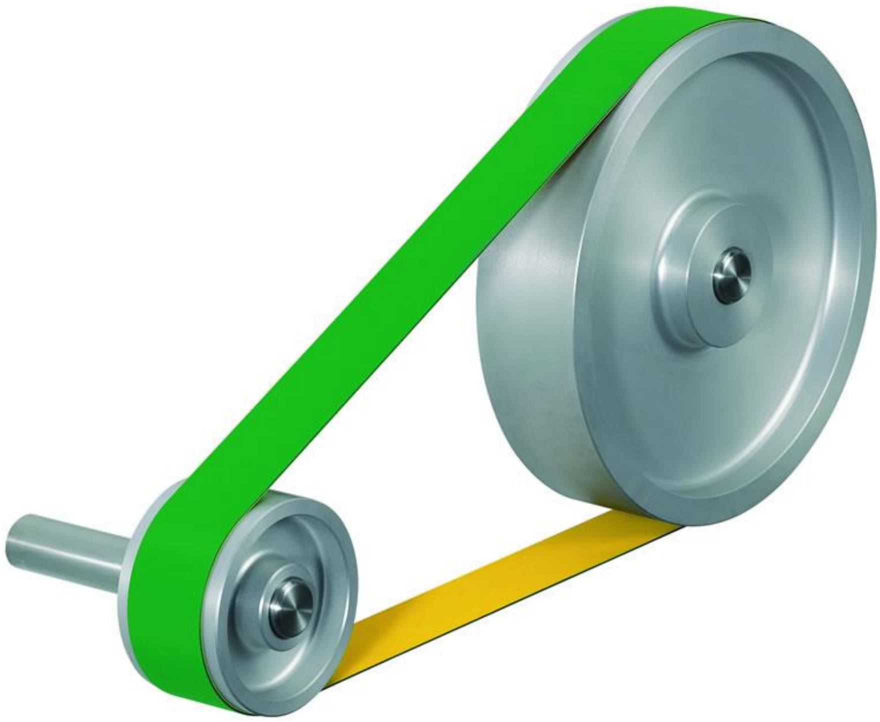
3.2: Types of Power Drives:

The following types of power drives are important from the subject point of view:

1. Belt Drives
2. Gear Drives
3. Chain Drives

Belt Drives:

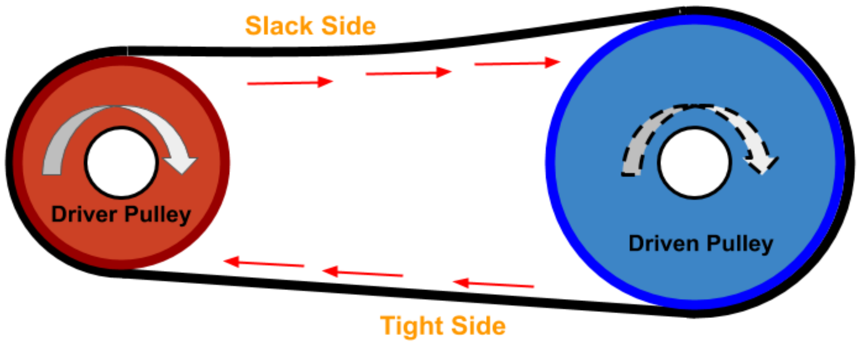
A belt drive is a mechanism, in which power is transmitted by the movement of a continuous flexible belt. It is used to transmit rotational motion from one shaft to another by means of pulleys which rotate at the same speed or at different speeds. A belt drive is found in almost every modern engine. The amount of power transmitted between the two is dependent on the amount of friction between the two shafts. The pulley, which transmits power is called **driving pulley or driver** and the pulley to which power is transmitted is called **driven pulley or follower**.

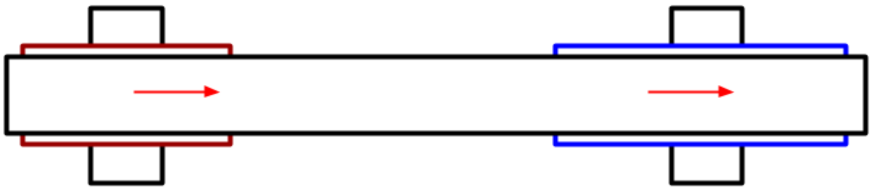


Types of Belt Drives:

The power from one pulley to another may be transmitted by any of the following types of belt drives:

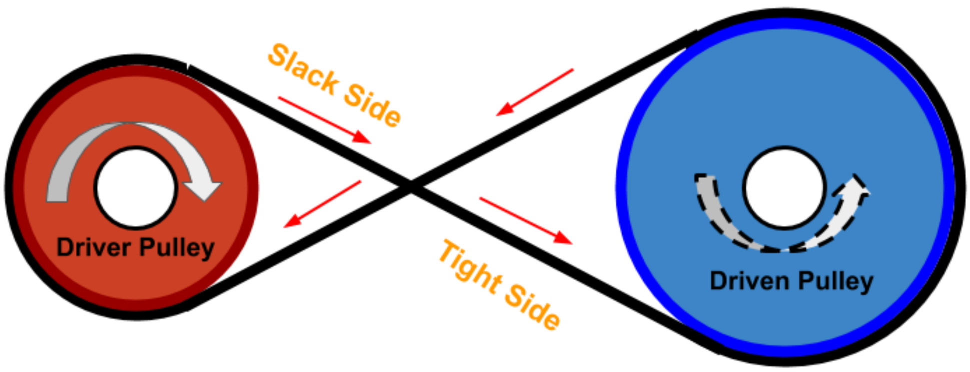
1. **Open belt drive:**

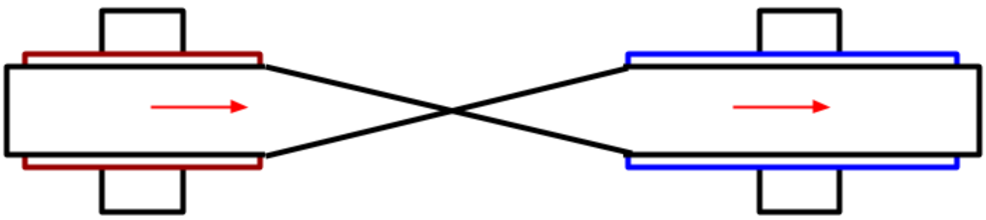




It is used with shafts arranged parallel and rotating in the same direction. In this case, the driver pulls the belt from one side (lower side) and delivers it to the other side (upper side). The lower side belt, because of more tension, is known as **tight side** whereas the upper side belt, because of less tension, is known as **slack side**.

2. **Crossed or twist belt drive:**





It is used with shafts arranged parallel and rotating in the opposite direction. In this case, the driver pulls the belt from one side (non-leading diagonal side) and delivers it to the other side (leading diagonal side). The non-leading diagonal side belt, because of more tension is known as **tight side** whereas the leading diagonal side belt, because of less tension, is known as **slack side**.

3.3: Velocity Ratio of Belt Drive:

It is the ratio between the velocities of the driver and the follower or driven.

Let

d_1 = Diameter of the driver

d_2 = Diameter of the driven

N_1 = Speed of the driver in rpm

N_2 = Speed of the driven in rpm

Length of the belt that passes over the driver in 1 minute = $\pi d_1 N_1$

Similarly, Length of the belt that passes over the driven in 1 minute = $\pi d_2 N_2$

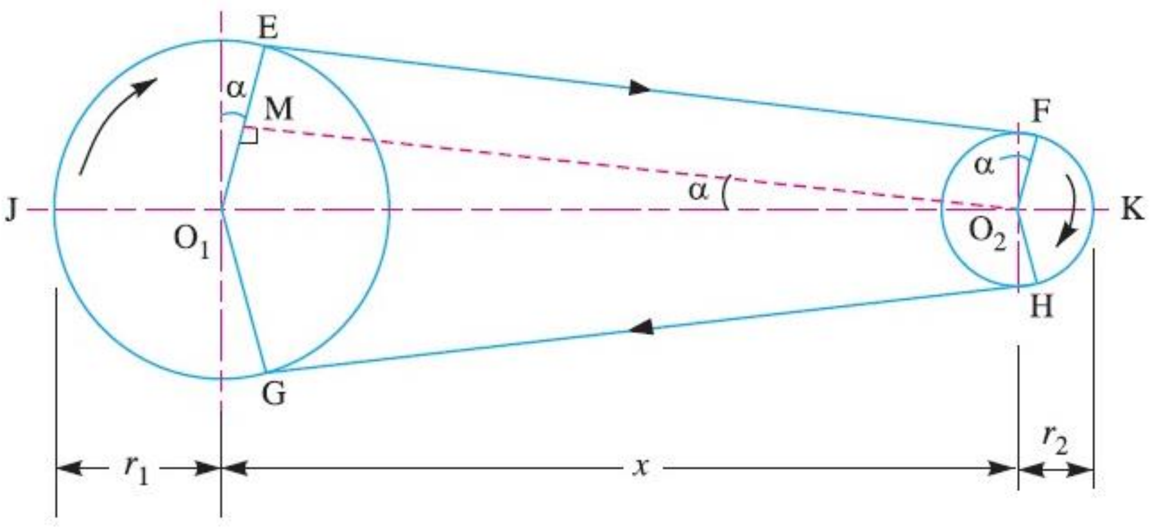
Since, the length of the belt that passes over the driver in 1 minute = Length of the belt that passes over the driven in 1 minute, therefore

$$\pi d_1 N_1 = \pi d_2 N_2$$

$$\text{Velocity Ratio, } \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

Length of the Belt:

I. For Open Belt Drive:



Let

r_1 = Radius of the Larger Pulley

r_2 = Radius of the Smaller Pulley

L = Total length of the belt

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in the figure.

Through O_2, draw O_2M parallel to FE.

From the geometry of the figure, we find that O_2M will be perpendicular to O_1E.

Let the $Angle\ MO_2O_1 = \alpha\ radians$

We know that the length of the belt,

$L = Arc\ GJE + EF + Arc\ FKH + HG = 2\,(Arc\ JE + EF + Arc\ FK)$ Eq^n (1)

From the geometry of the figure, we find that

$$Sin\ \alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E - EM}{O_1O_2} = \frac{r_1 - r_2}{x}$$

Since, α is very small, therefore putting:

$$Sin\ \alpha = \alpha = \frac{r_1-r_2}{x} \qquad \text{------ Eq}^n\ (2)$$

$$Arc\ JE = r_1\ \left(\frac{\pi}{2} + \alpha\right) \qquad \text{------ Eq}^n(3)$$

Similarly, $Arc\ FK = r_2\ \left(\frac{\pi}{2} - \alpha\right)$ ------ Eq^n(4)

And, $EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 - r_2)^2} = x\sqrt{1 - \left(\frac{r_1-r_2}{x}\right)^2}$

Expanding the above Equation by Binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1-r_2}{x}\right)^2 + \cdots\right] = x - \frac{(r_1-r_2)^2}{2x} \qquad \text{------ Eq}^n(5)$$

Substituting the values of Arc JE from Eq^n(3), Arc FK from Eq^n(4) and EF from Eq^n(5) in Eq^n(1), we get

$$L = 2\,(Arc\ JE + EF + Arc\ FK)$$

$$L = 2\left[r_1\ \left(\frac{\pi}{2} + \alpha\right) + r_2\ \left(\frac{\pi}{2} - \alpha\right) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= 2\left[r_1\frac{\pi}{2} + r_1\alpha + r_2\frac{\pi}{2} - r_2\alpha + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 - r_2) + x - \frac{(r_1 - r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

Substituting the values of $\alpha = \frac{r_1 - r_2}{x}$ from Eq^n(2), we get

$$L = \pi(r_1 + r_2) + 2\frac{(r_1 - r_2)}{x}(r_1 - r_2) + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2\frac{(r_1 - r_2)^2}{x} + 2x - \frac{(r_1 - r_2)^2}{x}$$

$$\therefore \boldsymbol{L = \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}}$$

II. For Cross Belt Drive:

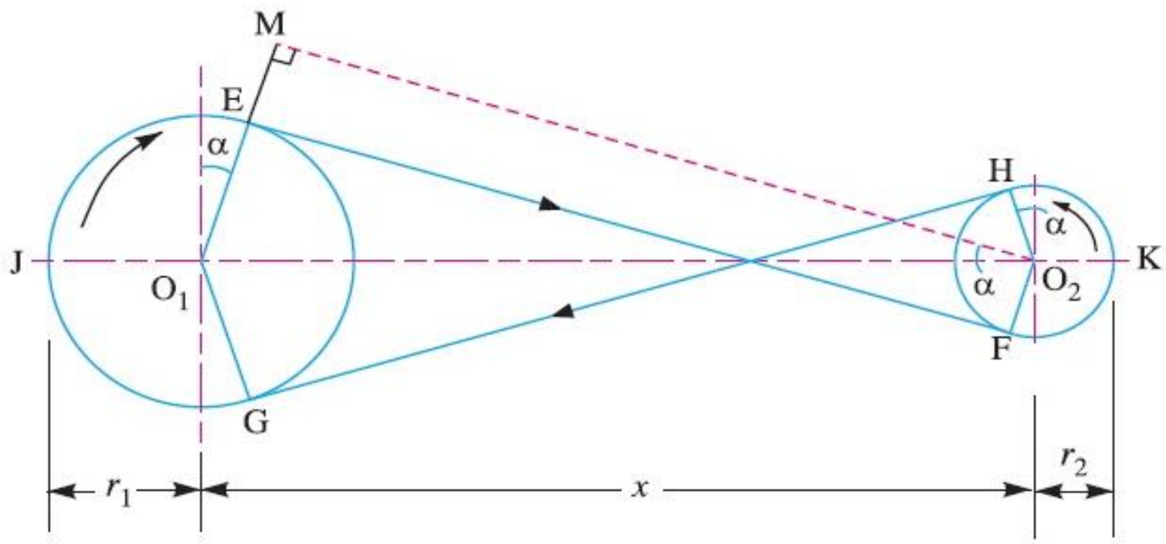
Let, r_1 = Radius of the Larger Pulley

r_2 = Radius of the Smaller Pulley

L = Total length of the belt

Let the belt leaves the larger pulley at E and G and the smaller pulley at F and H as shown in the figure.

Through O_2, draw O_2M parallel to FE.



From the geometry of the figure, we find that O_2M will be perpendicular to O_1E.

Let the $Angle\ MO_2O_1 = \alpha\ radians$

We know that the length of the belt,

$L = Arc\ GJE + EF + Arc\ FKH + HG = 2\,(Arc\ JE + EF + Arc\ FK)$ Eq^n (1)

From the geometry of the figure, we find that

$$Sin\ \alpha = \frac{O_1M}{O_1O_2} = \frac{O_1E + EM}{O_1O_2} = \frac{r_1 + r_2}{x}$$

Since, α is very small, therefore putting:

$$Sin\ \alpha = \alpha = \frac{r_1 + r_2}{x} \qquad \text{------ Eq}^n\ (2)$$

$$Arc\ JE = r_1\left(\frac{\pi}{2} + \alpha\right) \qquad \text{------ Eq}^n(3)$$

Similarly, $Arc\ FK = r_2\left(\frac{\pi}{2} + \alpha\right)$ ------ Eq^n(4)

And, $EF = MO_2 = \sqrt{(O_1O_2)^2 - (O_1M)^2} = \sqrt{x^2 - (r_1 + r_2)^2} = x\sqrt{1 - \left(\frac{r_1 + r_2}{x}\right)^2}$

Expanding the above Equation by Binomial theorem,

$$EF = x\left[1 - \frac{1}{2}\left(\frac{r_1 + r_2}{x}\right)^2 + \cdots\right] = x - \frac{(r_1 + r_2)^2}{2x} \qquad \text{------ Eq}^n(5)$$

Substituting the values of Arc JE from Eq^n(3), Arc FK from Eq^n(4) and EF from Eq^n(5) in Eq^n(1), we get

$$L = 2\,(Arc\ JE + EF + Arc\ FK)$$

$$L = 2\left[r_1\left(\frac{\pi}{2} + \alpha\right) + r_2\left(\frac{\pi}{2} + \alpha\right) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= 2\left[r_1\frac{\pi}{2} + r_1\alpha + r_2\frac{\pi}{2} + r_2\alpha + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= 2\left[\frac{\pi}{2}(r_1 + r_2) + \alpha(r_1 + r_2) + x - \frac{(r_1 + r_2)^2}{2x}\right]$$

$$= \pi(r_1 + r_2) + 2\alpha(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

Substituting the values of $\alpha = \frac{r_1 - r_2}{x}$ from Eq^n(2), we get

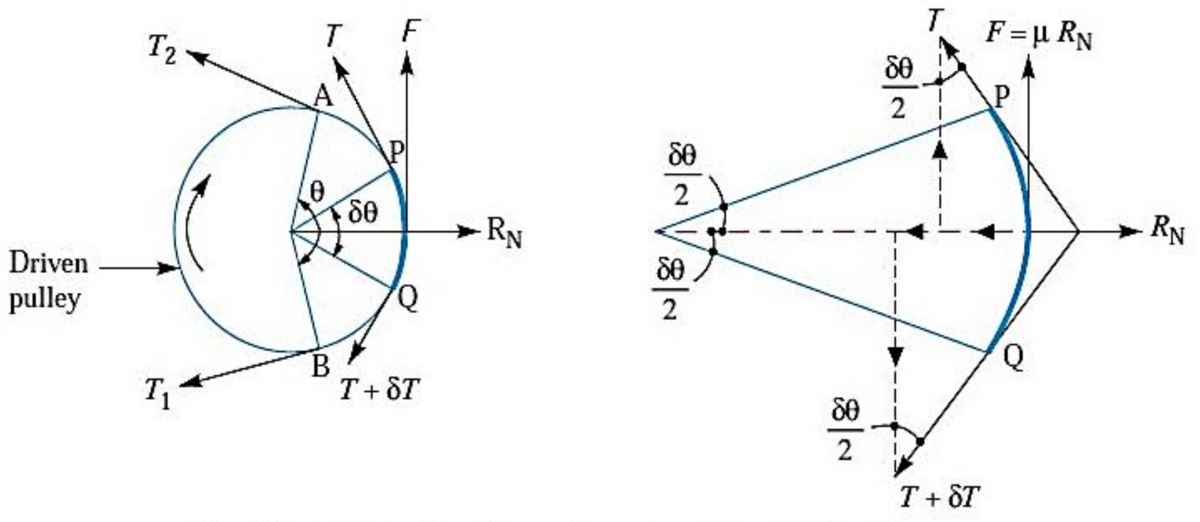
$$L = \pi(r_1 + r_2) + 2\frac{(r_1 + r_2)}{x}(r_1 + r_2) + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(r_1 + r_2) + 2\frac{(r_1 + r_2)^2}{x} + 2x - \frac{(r_1 + r_2)^2}{x}$$

$$\therefore \boldsymbol{L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}}$$

3.4: Ratio of Belt Tensions for Flat Belt Drive:

Consider a driven pulley rotating in the clock wise direction as shown in the figure.



Let

T_1 = Tension in the belt on the tight side

T_2 = Tension in the belt on the slack side

θ = Angle of contacts in radians

μ = Co-efficient of friction between the belt and pulley

Now consider a small portion of the belt PQ, subtending an angle $\delta\theta$ at the centre of the pulley as shown in the figure.

The belt PQ is in equilibrium under the following forces:

1. Tension T in the belt at P
2. Tension $T + \delta T$ in the belt at Q
3. Normal Reaction R_N
4. Frictional Force, $F = \mu R_N$

Resolving all the forces horizontally and equating the same,

$$R_N = (T + \delta T) \sin\frac{\delta\theta}{2} + T \sin\frac{\delta\theta}{2} \quad \text{------- Eq}^n(1)$$

Since, the angle $\delta\theta$ is very small, therefore putting $sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$ in Eq^n(1)

$$R_N = (T + \delta T)\frac{\delta\theta}{2} + T\frac{\delta\theta}{2} = \frac{T\delta\theta}{2} + \frac{\delta T\delta\theta}{2} + \frac{T\delta\theta}{2}$$

$$\boldsymbol{R_N = T.\delta\theta} \text{ neglecting the term } \frac{\delta T\delta\theta}{2} \quad \text{------- Eq}^n(2)$$

Now resolving the forces vertically, we have

$$\mu R_N = (T + \delta T) \cos\frac{\delta\theta}{2} - T \cos\frac{\delta\theta}{2} \quad \text{------- Eq}^n(3)$$

Since, the angle $\delta\theta$ is very small, therefore putting $cos\frac{\delta\theta}{2} = 1$ in Eq^n(3)

$$\mu R_N = (T + \delta T) - T = \delta T$$

$$\text{Or, } R_N = \frac{\delta T}{\mu} \quad \text{------- Eq}^n(4)$$

Equating the values of R_N from Eq^n(2) and Eq^n(4),

$$T.\delta\theta = \frac{\delta T}{\mu} \; or \; \frac{\delta T}{T} = \mu.\delta\theta$$

Integrating both sides between the limits T_2 to T_1 and from 0 to θ respectively.

$$\int_{T_2}^{T_1} \frac{\delta T}{T} = \mu \int_0^{\theta} \delta\theta \qquad or \;\; \log_e\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

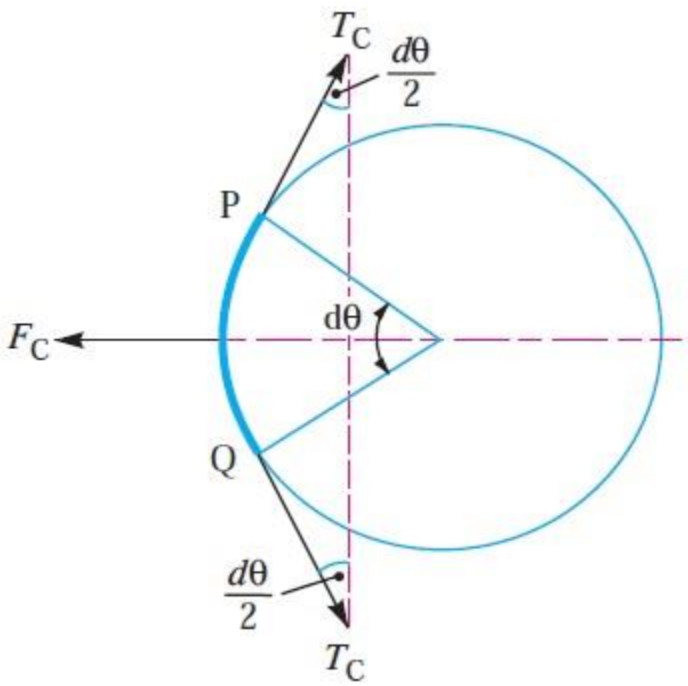
$$\therefore \boldsymbol{\frac{T_1}{T_2} = e^{\mu\theta}}$$

The above Equation can be expressed in terms of corresponding logarithm to the base 10, i.e.,

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta$$

Centrifugal Tension:

Since the belt is continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as slack sides. The tension caused by centrifugal force is called *Centrifugal Tension*.



Let

T_c = Centrifugal Tension in N

m = Mass of the belt per unit length in kg

v = Linear velocity of the belt in m/s

r = Radius of the pulley over which the belt runs in m

Now consider a small portion of the belt PQ, subtending an angle $d\theta$ at the centre of the pulley as shown in the figure.

We know that the length of the belt PQ = $r.d\theta$

Mass of the belt PQ = $m.r.d\theta$

Then, Centrifugal Force acting on the belt PQ ,

$$F_c = (m.r.d\theta)\frac{v^2}{r} = m.d\theta.v^2$$

The centrifugal tension T_c acting tangentially at P & Q keeps the belt in equilibrium.

Now resolving the forces horizontally and equating the same, we have

$$T_c \sin\left(\frac{d\theta}{2}\right) + T_c \sin\left(\frac{d\theta}{2}\right) = F_c = m.d\theta.v^2$$

Since, the angle $d\theta$ is very small, therefore putting $sin\frac{d\theta}{2} = \frac{d\theta}{2}$ in the above expression.

$$2T_c \left(\frac{d\theta}{2}\right) = m.d\theta.v^2 \qquad or \qquad T_c = m.v^2$$

Little Further:

1. When the centrifugal tension is taken into account, then the total tension in the tight side, $T_{t1} = T_1 + T_c$.

   And Total Tension in slack side, $T_{t2} = T_2 + T_c$.

2. Power transmitted, $P = (T_{t1} - T_{t2})\,v = (T_1 - T_2)\,v$ in watts

   Thus, we see that centrifugal tension has no effect on the power transmitted.

3. The ratio of driving tensions may be written as

$$\boldsymbol{\frac{T_{t1} - T_c}{T_{t2} - T_c} = e^{\mu\theta}}$$

Initial Tension:

When the belt is wound round the two pulleys, its two ends are joined together; so that the belt may continuously move over the pulleys, since the motion of the belt from the driver and the follower is governed by a firm grip, due to friction between the belt and the pulleys. In order to increase this grip, the belt is tightened up. At this stage, even when the pulleys are stationary, the belt is subjected to some tensions called initial tension.

Let

T_0 = Initial Tension in the belt

T_1 = Tension in the tight side of the belt

T_2 = Tension in the slack side of the belt

α = co-efficient of increase of belt length per unit force

A little consideration will show that the increase of tension in the tight side = $T_1 - T_0$

And, increase in the length of the belt on the tight side = $\alpha\ (T_1 - T_0)$

Similarly, decrease in the length of the belt on the slack side = $\alpha\ (T_0 - T_2)$

Assuming that the belt material is perfectly elastic such that the length of the belt remains constant, when it is at rest or in motion, therefore increase in length on the tight side is equal to decrease in the length on the slack side.

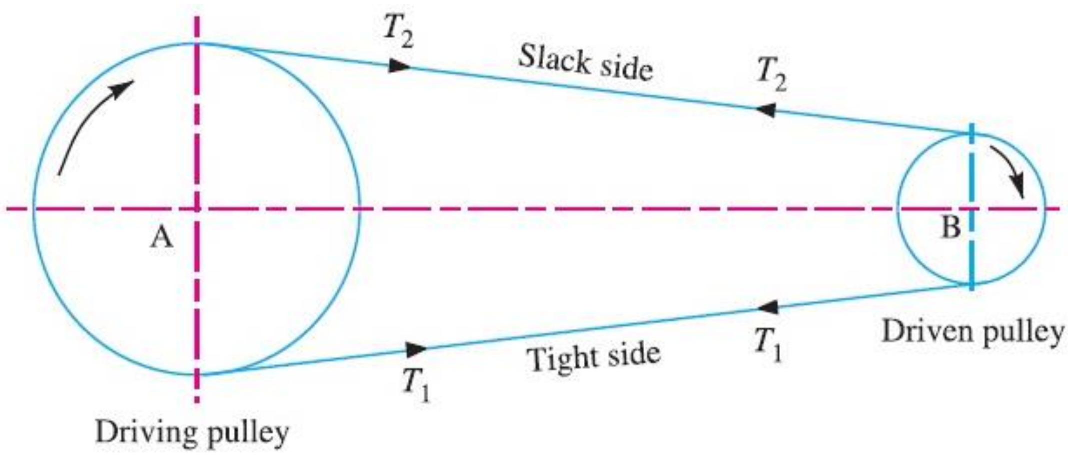
Thus, equating the equations

$$\alpha\ (T_1 - T_0) = \alpha\ (T_0 - T_2)$$

$$\text{Or, } T_1 - T_0 = T_0 - T_2$$

$$\text{Or, } \boldsymbol{T_0 = \frac{T_1+T_2}{2} = \frac{T_1+T_2+2T_c}{2}}$$

3.5: Power Transmitted by Belt:



The above figure shows the driving pulley (or driver) A and the driven pulley (or follower) B. We have already discussed that the driving pulley pulls the belt from one side and delivers the same to the other side. It is thus obvious that the tension on the former side (i.e. tight side) will be greater than the latter side (i.e. slack side).

Let

T_1 = Tension in the tight side of the belt in N

T_2 = Tension in the slack side of the belt in N

r_1 = Radius of the Driver in m

r_2 = Radius of the Driven or follower in m

v = Linear velocity of the belt in m/s

The effective turning (driving) force at the circumference of the follower is the difference between the two tensions (i.e., $T_1 - T_2$).

∴ Work done per second = $(T_1 - T_2)\, v$ N-m/s

and power transmitted, P = $(T_1 - T_2)\, v$ W ... (∵ 1 N-m/s = 1 W)

A little consideration will show that the torque exerted on the driving pulley is $(T_1 - T_2)\, r_1$. Similarly, the torque exerted on the driven pulley i.e., follower is $(T_1 - T_2)\, r_2$.

Numerical No.- 01

Find the power transmitted by a belt running over a pulley of 600 mm diameter at 200 r.p.m. The coefficient of friction between the belt and the pulley is 0.25, angle of lap 160° and maximum tension in the belt is 2500 N.

Solution:

Given: d = 600 mm = 0.6 m ; N = 200 rpm ; μ = 0.25 ; $\theta = 160^o = \frac{160\times\pi}{180}$ = 2.793 rad ; T_1 = 2500 N

We know that velocity of the belt,

$$v = \frac{\pi dN}{60} = \frac{\pi \times 0.6 \times 200}{60} = 6.284\ m/s$$

Again, we know that

$$2.3\ \log\left(\frac{T_1}{T_2}\right) = \mu\ .\ \theta = 0.25 \times 2.793 = 0.6982$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.6982}{2.3} = 0.3036$$

$$\frac{T_1}{T_2} = 2.01$$

And

$$T_2 = \frac{T_1}{2.01} = \frac{2500}{2.01} = 1244\ \mathrm{N}$$

We know that the power transmitted by the belt

$P = (T_1 - T_2)\ v = (2500 - 1244)\ 6.284 = 7890\ \mathrm{W} = 7.89\ \mathrm{kW}$ **(Ans)**

Numerical No.- 02

A casting weighing 9 kN hangs freely from a rope which makes 2.5 turns round a drum of 300 mm diameter revolving at 20 r.p.m. The other end of the rope is pulled by a man. The coefficient of friction is 0.25. Determine 1. The force required by the man, and 2. The power to raise the casting.

Solution:

Given: W = T_1 = 9 kN = 9000 N ; d = 300 mm = 0.3 m ; N = 20 rpm ; μ = 0.25

1. **Force required by the man:**

   Let T_2 = Force required by the man

   Since the rope makes 2.5 turns round the drum, therefore angle of contact, $\theta = 2.5 \times 2\pi = 5\pi\ \mathrm{rad}$

   We know that

$$2.3\ \log\left(\frac{T_1}{T_2}\right) = \mu\ .\ \theta = 0.25 \times 5\pi = 3.9275$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{3.9275}{2.3} = 1.71$$

$$\frac{T_1}{T_2} = 51$$

$$T_2 = \frac{T_1}{51} = \frac{9000}{51} = 176.47 \text{ N}$$

2. **Power to raise the casting:**

We know that, velocity of the rope

$$v = \frac{\pi d N}{60} = \frac{\pi \times 0.3 \times 20}{60} = 0.3142\ m/s$$

Power to raise the casting

$$P = (T_1 - T_2)\ v = (9000 - 176.47)\ 0.3142 = 2772 \text{ W} = 2.772 \text{ kW} \quad \textbf{(Ans)}$$

3.6: The belt thickness and width for permissible stress for open and cross belt drive considering centrifugal tension:

A little consideration will show that the maximum tension in the belt (T) is equal to the total tension in the tight side of the belt (T_{t1}).

Let

σ = Maximum safe stress in N/mm^2,

b = Width of the belt in mm

t = Thickness of the belt in mm

We know that maximum tension in the belt,

T = Maximum stress × cross-sectional area of belt = σ. b. t

When centrifugal tension is neglected, then T (or T_{t1}) = T_1, i.e., Tension in the tight side of the belt and when centrifugal tension is considered, then T (or T_{t1}) = T_1 + T_C

Numerical No.- 03

A leather belt is required to transmit 7.5 kW from a pulley 1.2 m in diameter, running at 250 r.p.m. The angle embraced is 165° and the coefficient of friction between the belt and the pulley is 0.3. If the safe working stress for the leather belt is 1.5 MPa, density of leather 1 Mg/m^3 and thickness of belt 10 mm, determine the width of the belt taking centrifugal tension into account.

Solution:

Given: $P = 7.5$ kW $= 7500$ W ; $d = 1.2$ m ; $N = 250$ r.p.m. ; $\theta = 165 = 165 \times \pi / 180 = 2.88$ rad ; $\mu = 0.3$; $\sigma = 1.5$ MPa $= 1.5 \times 10^6$ N/m^2 ; $\rho = 1$ Mg/m^3 $= 1 \times 106$ $g/m^3 = 1000$ kg/m^3; $t = 10$ mm $= 0.01$ m

Let

b = Width of belt in meters,

T_1 = Tension in the tight side of the belt in N

T_2 = Tension in the slack side of the belt in N

We know that velocity of the belt,

$$v = \pi\, d\, N / 60 = \pi \times 1.2 \times 250/60 = 15.71 \text{ m/s}$$

and power transmitted (P),

$$7500 = (T_1 - T_2)\, v = (T_1 - T_2)\, 15.71$$

$$\therefore T_1 - T_2 = 7500 / 15.71 = 477.4 \text{ N} \qquad \text{...(i)}$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu\,.\,\theta = 0.3 \times 2.88 = 0.864$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.864}{2.3} = 0.3756$$

$$\frac{T_1}{T_2} = 2.375 \qquad \text{... (ii)}$$

From equations (i) and (ii),

$$T_1 = 824.6 \text{ N, and } T_2 = 347.2 \text{ N}$$

We know that mass of the belt per meter length,

$$m = \text{Area} \times \text{length} \times \text{density} = b.t.l.\rho$$

$$= b \times 0.01 \times 1 \times 1000 = 10\, b \text{ kg}$$

$\therefore$ Centrifugal tension,

$$T_C = m.\, v^2 = 10\, b\, (15.71)^2 = 2468\, b \text{ N}$$

and maximum tension in the belt,

$$T = \sigma . b . t = 1.5 \times 10^6 \times b \times 0.01 = 15\,000\,b\text{ N}$$

We know that,

$$T = T_1 + T_C$$

$$\text{or } 15000\,b = 824.6 + 2468\,b$$

$$\text{or } 15\,000\,b - 2468\,b = 824.6$$

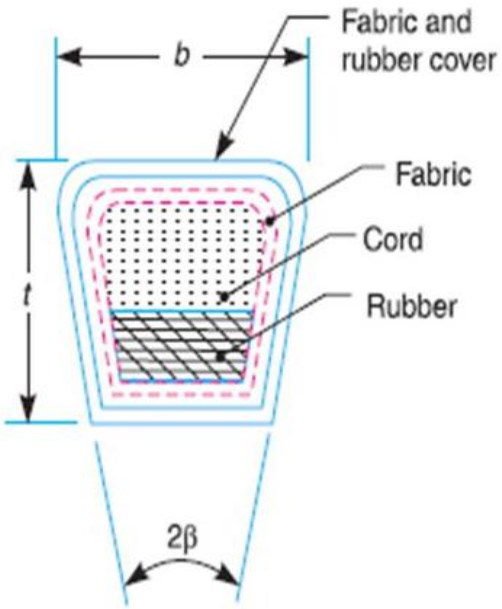
$$\text{or } 12\,532\,b = 824.6$$

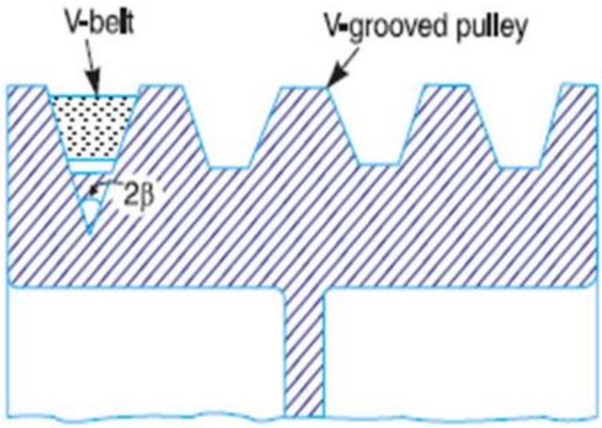
$$\therefore b = 824.6 / 12532 = 0.0658\text{ m} = 65.8\text{ mm (Ans)}$$

3.7: V Belt & V Belt Pulleys:

A V-belt is mostly used in factories and workshops where a great amount of power is to be transmitted from one pulley to another when the two pulleys are very near to each other.

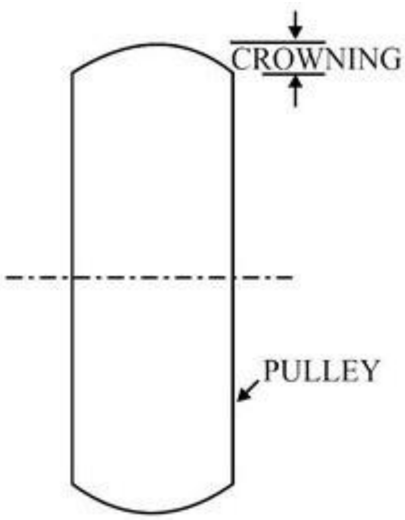
The V-belts are made of fabric and cords moulded in rubber and covered with fabric and rubber, as shown in Figure. These belts are moulded to a trapezoidal shape and are made endless. These are particularly suitable for short drives i.e. when the shafts are at a short distance apart. In case of flat belt drive, the belt runs over the pulleys whereas in case of V-belt drive, the rim of the pulley is grooved in which the V-belt runs. The effect of the groove is to increase the frictional grip of the V-belt on the pulley and thus to reduce the tendency of slipping. In order to have a good grip on the pulley, the V-belt is in contact with the side faces of the groove and not at the bottom. The power is transmitted by the wedging action between the belt and the V-groove in the pulley.





A clearance must be provided at the bottom of the groove, as shown in Figure, in order to prevent touching to the bottom as it becomes narrower from wear. In order to increase the power output, several V- belts may be operated side by side. It may be noted that in multiple V-belt drive, all the belts should stretch at the same rate so that the load is equally divided between them. When one of the set of belts break, the entire set should be replaced at the same time. If only one belt is replaced, the new unworn and unstressed belt will be more tightly stretched and will move with different velocity.

3.8: Crowning of Pulleys:



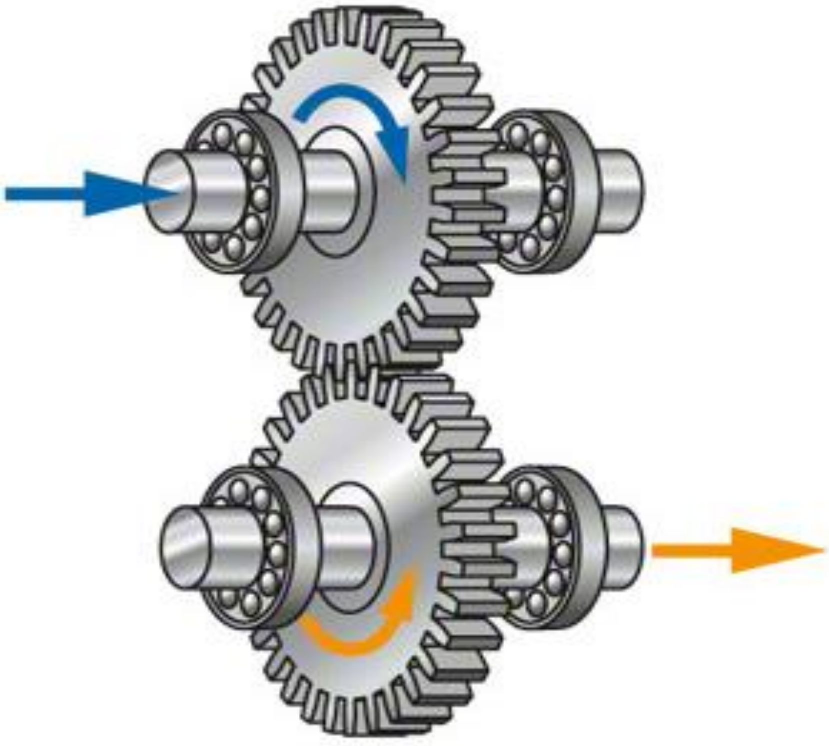
A crowned pulley is a pulley that has a slight hump in the middle, tapering off ever so slightly towards either edge.

A crowned pulley is used to increase the stability of the belt on the pulley and to run at the center of the pulley without slipping off from the edges.

CHAPTER NO.- 03 (Part- II)

POWER TRANSMISSION

Article No.	Name of the Article
3.9	Gear drives and its terminology.
3.10	Gear trains, Working principle of simple gear trains.
	Working principle of compound gear trains.
	Working principle of reverted gear trains.
	Working principle of epicyclic gear trains.
	Numerical problem
	Possible Questions



3.9: Gear Drives:

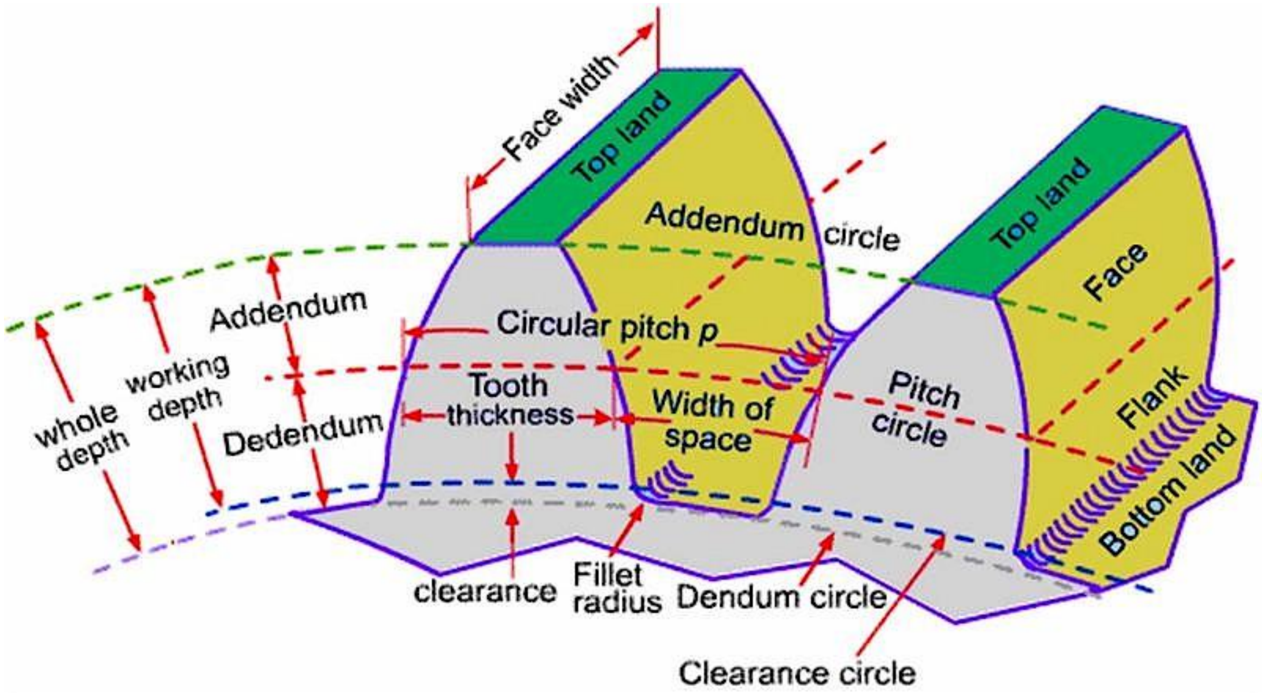
Sometimes, two or more gears are made to mesh with each other to transmit power from one shaft to another. Such a combination is called gear train or train of toothed wheels. The nature of the train used depends upon the velocity ratio required and the relative position of the axes of shafts. A gear train may consist of spur, bevel or spiral gears.

In precision machines, in which a definite velocity ratio is of importance (as in watch mechanism), the only positive drive is by means of gears or toothed wheels. A gear drive is also provided, when the distance between the driver and the follower is very small.

Important Terms Connected with Gear Drive:

1. ***Pitch Circle***: It is an imaginary circle which by pure rolling action, would give the same motion as the actual gear.

2. ***Pitch Circle Diameter***: It is the diameter of the pitch circle. The size of the gear is usually specified by the pitch circle diameter. It is also known as pitch diameter.
3. ***Addendum***: It is the radial distance of a tooth from the pitch circle to the top of the tooth.



4. ***Dedendum***: It is the radial distance of a tooth from the pitch circle to the bottom of the tooth.
5. ***Circular Pitch***: It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c.
   Mathematically,
   $$p_c = \frac{\pi D}{T}$$
   where, D = Diameter of the pitch circle, and T = Number of teeth on the wheel.
6. ***Diametral Pitch***: It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by p_d.
   Mathematically,
   $$p_d = \frac{T}{D} = \frac{\pi}{p_c}$$

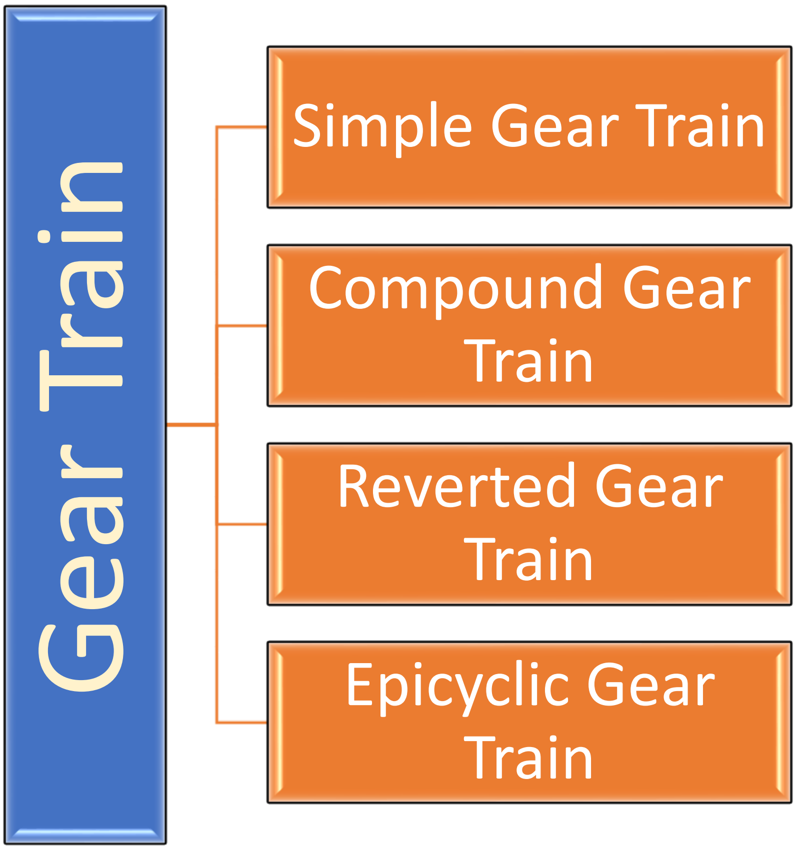
7. ***Module***: It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by m.
Mathematically,

$$m = \frac{D}{T} = \frac{1}{p_d}$$

3.10: Gear Trains:

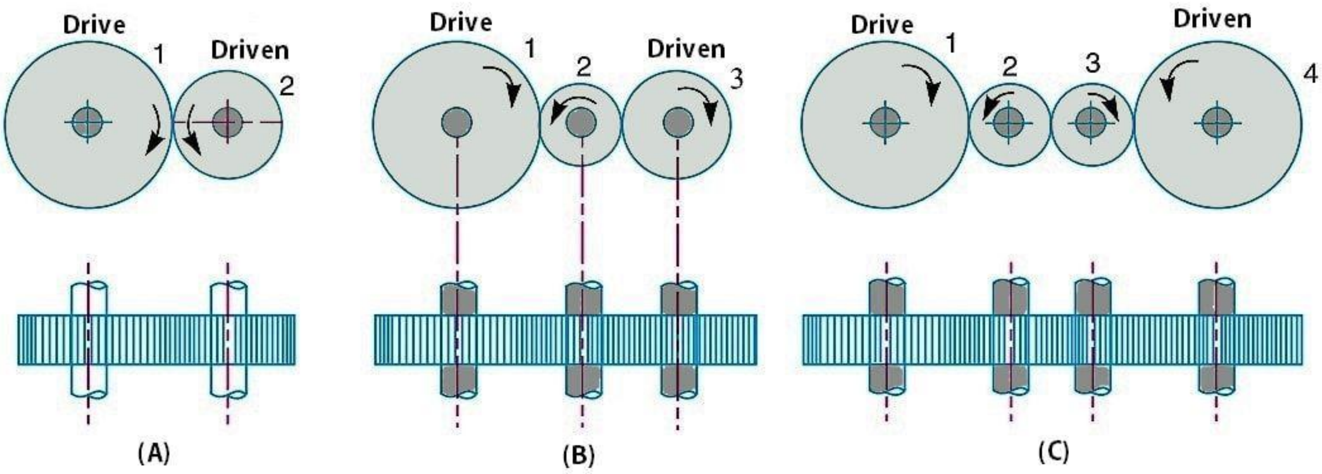
The combination of gear wheels or gears, used to increase or decrease the speed of the driven shaft is called gear train.

Types of Gear Trains:



Simple Gear Train:

When there is only one gear on each shaft, it is known as simple gear train. The gears are represented by their pitch circles.



When the distance between the two shafts is small, the two gears 1 and 2 are made to mesh with each other to transmit motion from one shaft to the other, as shown in Figure (A). Since the gear 1 drives the gear 2, therefore gear 1 is called the driver and the gear 2 is called the driven or follower. It may be noted that the motion of the driven gear is opposite to the motion of driving gear.

Let,

N_1 = Speed of Gear 1 (Driver) in rpm

N_2 = Speed of Gear 2 (Driven/ Follower) in rpm

T_1 = Number of Teeth on gear 1

T_2 = Number of Teeth on gear 2

Since the speed ratio (or velocity ratio) of gear train is the ratio of the speed of the driver to the speed of the driven or follower and ratio of speeds of any pair of gears in mesh is the inverse of their number of teeth, therefore

$$Speed\ Ratio = \frac{N_1}{N_2} = \frac{T_2}{T_1}$$

It may be noted that ratio of the speed of the driven or follower to the speed of the driver is known as train value of the gear train. Mathematically,

$$Train\ Value = \frac{N_2}{N_1} = \frac{T_1}{T_2}$$

From above, we see that the train value is the reciprocal of speed ratio.

Sometimes, the distance between the two gears is large. The motion from one gear to another, in such a case, may be transmitted by either of the following two methods:

1. By providing the large sized gear, or
2. By providing one or more intermediate gears

A little consideration will show that the former method (i.e. providing large sized gears) is very inconvenient and uneconomical method; whereas the latter method (i.e. providing one or more intermediate gear) is very convenient and economical.

It may be noted that when the number of intermediate gears are **odd**, the motion of both the gears (i.e. driver and driven or follower) is **like**.

But if the number of intermediate gears are **even**, the motion of the driven or follower will be in the **opposite** direction of the driver.

Now consider a simple train of gears with one intermediate gear as shown in Figure **B.**

Let

N_1 = Speed of driver in r.p.m.,

N_2 = Speed of intermediate gear in r.p.m.

N_3 = Speed of driven or follower in r.p.m.,

T_1 = Number of teeth on driver,

T_2 = Number of teeth on intermediate gear, and

T_3 = Number of teeth on driven or follower.

Since the driving gear 1 is in mesh with the intermediate gear 2, therefore speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1}$$

Similarly, as the intermediate gear 2 is in mesh with the driven gear 3, therefore speed ratio for these two gears is

$$\frac{N_2}{N_3} = \frac{T_3}{T_2}$$

The speed ratio of the gear train as shown in Fig. 13.1 (b) is obtained by multiplying the above equations

$$\frac{N_1}{N_2} \times \frac{N_2}{N_3} = \frac{T_2}{T_1} \times \frac{T_3}{T_2}$$

$$\frac{N_1}{N_3} = \frac{T_3}{T_1}$$

Hence, $\text{Speed Ratio} = \frac{\textit{Speed of Driver}}{\textit{Speed of Driven}} = \frac{\textit{No. of teeth on Driven}}{\textit{No. of Teeth on Driver}}$

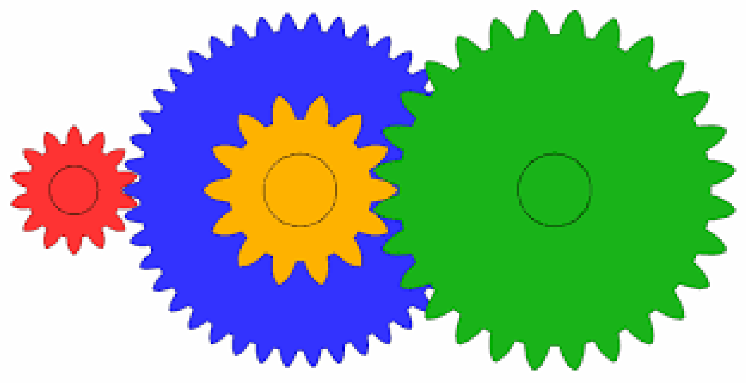
And $\text{Train Value} = \frac{\textit{Speed of Driven}}{\textit{Speed of Driver}} = \frac{\textit{No. of teeth on Driver}}{\textit{No. of Teeth on Driven}}$

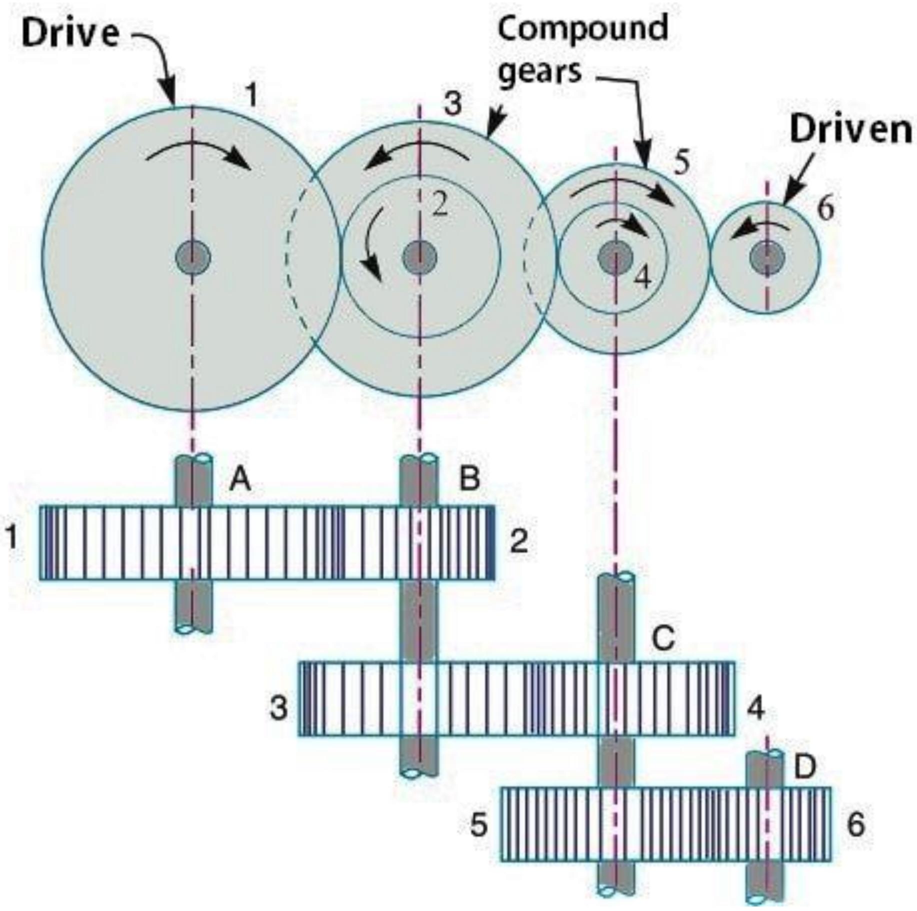
From above, we see that the speed ratio and the train value, in a simple gear train, is independent of the size and number of intermediate gears. These intermediate gears are called ***idle gears***. The idle gears are used for the following two purposes:

1. To connect gears where a large centre distance is required
2. To obtain the desired direction of motion of the driven gear (i.e., clockwise or anticlockwise).

Compound Gear Train:

When there is more than one gear on a shaft, it is called a compound train of gear or compound gear train.





In the figure, the gear 1 is the driving gear mounted on shaft A, gears 2 and 3 are compound gears which are mounted on shaft B. The gears 4 and 5 are also compound gears which are mounted on shaft C and the gear 6 is the driven gear mounted on shaft D.

Let

N_1 = Speed of driving gear 1 in r.p.m.,

T_1 = Number of teeth on driving gear 1,

N_2 , N_3 …. N_6 = Number of teeth on respective gears,

T_2 , T_3 …. T_6 = Number of teeth on respective gears.

Since the gear 1 is in mesh with the gear 2, therefore speed ratio for these two gears is

$$\frac{N_1}{N_2} = \frac{T_2}{T_1} \qquad \dots \dots (1)$$

Similarly, for gears 3 and 4, speed ratio is

$$\frac{N_3}{N_4} = \frac{T_4}{T_3} \qquad \dots \dots (2)$$

and for gears 5 and 6, speed ratio is

$$\frac{N_5}{N_6} = \frac{T_6}{T_5} \qquad \dots \dots (3)$$

The speed ratio of compound gear train is obtained by multiplying the equations (1) , (2) & (3). Then we get

$$\frac{N_1}{N_2} \times \frac{N_3}{N_4} \times \frac{N_5}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5}$$

Since gears 2 and 3 are mounted on one shaft B, therefore $N_2 = N_3$. Similarly gears 4 and 5 are mounted on shaft C, therefore $N_4 = N_5$. Then

$$\frac{N_1}{N_6} = \frac{T_2}{T_1} \times \frac{T_4}{T_3} \times \frac{T_6}{T_5} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

Hence,

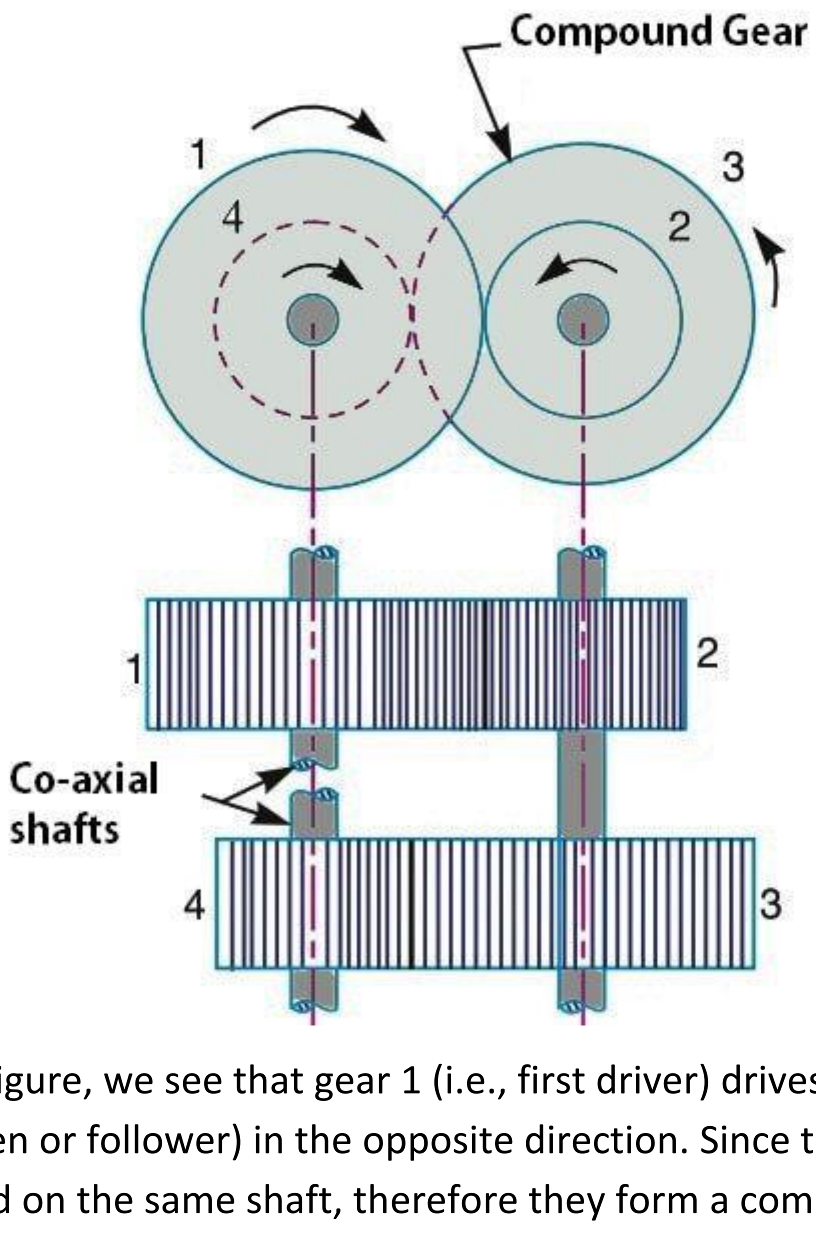
$$\text{Speed Ratio} = \frac{\textit{Speed of the first Driver}}{\textit{Speed of the last Driven}} = \frac{\textit{Product of no. of teeth on Drivens}}{\textit{Product of no. of Teeth on Drivers}}$$

$$\text{Train Value} = \frac{\textit{Speed of the last Driven}}{\textit{Speed of the first Driver}} = \frac{\textit{Product of no. of teeth on Drivers}}{\textit{Product of no. of Teeth on Drivens}}$$

The advantage of a compound train over a simple gear train is that a ***much larger speed reduction*** from the first shaft to the last shaft can be obtained with small gears.

Reverted Gear Train:

When the axes of the first gear (i.e. first driver) and the last gear (i.e. last driven or follower) are co-axial, then the gear train is known as ***reverted gear train***.



In the above figure, we see that gear 1 (i.e., first driver) drives the gear 2 (i.e., first driven or follower) in the opposite direction. Since the gears 2 and 3 are mounted on the same shaft, therefore they form a compound gear and the gear 3 will rotate in the same direction as that of gear 2. The gear 3 (which is now the second driver) drives the gear 4 (i.e., the last driven or follower) in the same direction as that of gear 1. Thus, we see that in a reverted gear train, the motion of the first gear and the last gear is like.

Let

N_1 = Speed of driving gear 1,

T_1 = Number of teeth on driving gear 1,

r_1 = Pitch circle radius of driving gear 1

N_2 , N_3 & N_4 = Number of teeth on respective gears,

T_2 , T_3 & T_4 = Number of teeth on respective gears

r_2 , r_3 & r_4= Pitch circle radius of respective gears

Since the distance between the centres of the shafts of gears 1 and 2 as well as gears 3 and 4 is same, therefore

$$r_1 + r_2 = r_3 + r_4 \quad \ldots .(1)$$

Also, the circular pitch or module of all the gears is assumed to be same, therefore number of teeth on each gear is directly proportional to its circumference or radius. Therefore,

$$T_1 + T_2 = T_3 + T_4 \quad \ldots .(2)$$

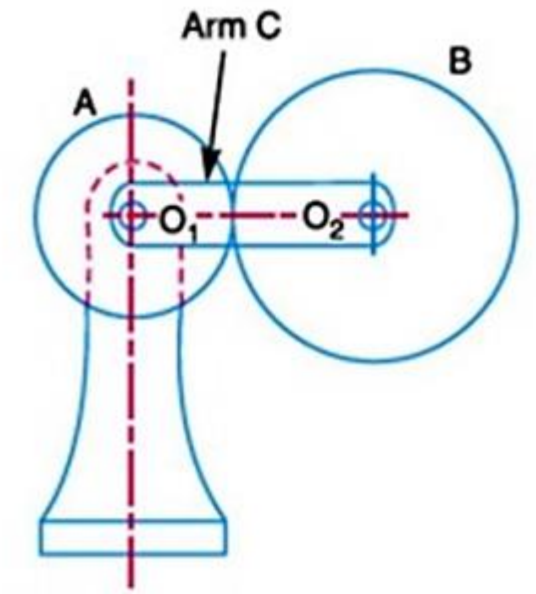
And, $$Speed\ Ratio = \frac{Product\ of\ no.\ of\ teeth\ on\ Drivens}{Product\ of\ no.\ of\ Teeth\ on\ Drivers}$$

$$\frac{N_1}{N_4} = \frac{T_2 \times T_4}{T_1 \times T_3} \quad \ldots ..(3)$$

From equations (1), (2) and (3), we can determine the number of teeth on each gear for the given centre distance, speed ratio and module only when the number of teeth on one gear is chosen arbitrarily.

The reverted gear trains are used in automotive transmissions, lathe back gears, industrial speed reducers, and in clocks (where the minute and hour hand shafts are co-axial).

Epicyclic Gear Train:



In an epicyclic gear train, the axes of the shafts on which the gears are mounted may move relative to a fixed axis.

A simple epicyclic gear train, as shown in above, where a gear A and the arm C have a common axis at O_1 about which they can rotate. The gear B meshes with gear A and has its axis on the arm at O_2 , about which the gear B can rotate. If the arm is fixed, the gear train is simple and gear A can drive gear B or vice- versa, but if gear A is fixed and the arm is rotated about the axis of gear A (i.e., O_1), then the gear B is forced to rotate upon and around gear A. Such a motion is called ***epicyclic*** and the gear trains arranged in such a manner that one or more of their members move upon and around another member are known as ***epicyclic gear trains*** (***epi*** means upon and ***cyclic*** means around). The epicyclic gear trains may be simple or compound.

The epicyclic gear trains are useful for transmitting high velocity ratios with gears of moderate size in a comparatively lesser space. The epicyclic gear trains are used in the back gear of lathe, differential gears of the automobiles, hoists, pulley blocks, wrist watches etc.

Velocity ratio of Epicyclic Gear Train:

Let

T_A = Speed of driving gear 1,

T_B = Number of teeth on driving gear 1,

First of all, let us suppose that the arm is fixed. Therefore, the axes of both the gears are also fixed relative to each other. When the gear A makes one revolution anticlockwise, the gear B will make T_A / T_B revolutions, clockwise. Assuming the anticlockwise rotation as positive and clockwise as negative, we may say that when gear A makes + 1 revolution, then the gear B will make (– T_A / T_B) revolutions. This statement of relative motion is entered in the first row of the table (as shown in table below).

Secondly, if the gear A makes + x revolutions, then the gear B will make (– x × T_A / T_B) revolutions. This statement is entered in the second row of the table. In other words, multiply each motion (entered in the first row) by x.

Thirdly, each element of an epicyclic train is given + y revolutions and entered in the third row. Finally, the motion of each element of the gear train is added up and entered in the fourth row.

Step No.	Conditions of motion	Revolutions of motion		
		Arm C	Gear A	Gear B
1	Arm fixed-gear A rotates through + 1 revolution i.e., 1 rev. anticlockwise	0	$+1$	$\frac{-T_A}{T_B}$
2	Arm fixed-gear A rotates through + x revolutions	0	$+x$	$\frac{-x \times T_A}{T_B}$
3	Add + y revolutions to all elements	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y-\frac{x \times T_A}{T_B}$

A little consideration will show that when two conditions about the motion of rotation of any two elements are known, then the unknown speed of the third element may be obtained by substituting the given data in the third column of the fourth row.

Numerical No.- 04

In an epicyclic gear train, an arm carries two gears A and B having 36 and 45 teeth respectively. If the arm rotates at 150 r.p.m. in the anticlockwise direction about the center of the gear A which is fixed, determine the speed of gear B. If the gear A instead of being fixed, makes 300 r.p.m. in the clockwise direction, what will be the speed of gear B?

Solution:

Given: T_A = 36 ; T_B = 45 ; N_C = 150 r.p.m. (anticlockwise)

First of all prepare the table of motions as given below:

Step No.	Conditions of motion	Revolutions of motion		
		Arm C	Gear A	Gear B
1	Arm fixed-gear A rotates through + 1 revolution i.e., 1 rev. anticlockwise	0	$+1$	$\frac{-T_A}{T_B}$
2	Arm fixed-gear A rotates through + x revolutions	0	$+x$	$\frac{-x \times T_A}{T_B}$
3	Add + y revolutions to all elements	$+y$	$+y$	$+y$
4	Total motion	$+y$	$x+y$	$y - \frac{x \times T_A}{T_B}$

Speed of gear B when gear A is fixed:

Since the speed of arm is 150 r.p.m. anticlockwise, therefore from the fourth row of the table, y = + 150 r.p.m

Also, the gear A is fixed, therefore, x + y = 0 or x = – y = – 150 r.p.m.

∴ Speed of gear B,

$$N_B = y - \frac{x \times T_A}{T_B} = 150 + \frac{150 \times 36}{45} = +270\,rpm$$

Therefore, speed of Gear B is **270 rpm** anticlockwise. **(*Ans*)**

Speed of gear B when gear A makes 300 r.p.m. clockwise:

Since the gear A makes 300 r.p.m. clockwise, therefore from the fourth row of the table; x + y = – 300 or x = – 300 – y = – 300 – 150 = – 450 r.p.m

∴ Speed of gear B,

$$N_B = y - \frac{x \times T_A}{T_B} = 150 + \frac{450 \times 36}{45} = +510\,rpm$$

Therefore, speed of Gear B is **510 rpm** anticlockwise. **(*Ans*)**

Short Type Questions with Answer

Question No.-01:

What do you understand by (mechanical) power transmission?

Ans: Mechanical power transmission is the transfer of energy from where it's generated to a place where it is used to perform work using simple machines, linkages and mechanical power transmission elements.

Question No.-02:

What is belt drive?

Ans: A belt drive is a mechanism, in which power is transmitted by the movement of a continuous flexible belt. It is used to transmit rotational motion from one shaft to another by means of pulleys which rotate at the same speed or at different speeds.

Question No.-03:

Define velocity ratio of belt drive.

Ans: It is the ratio between the velocities of the driver and the follower or driven.

Mathematically, Velocity Ratio, $\frac{N_2}{N_1} = \frac{d_1}{d_2}$

Question No.-04:

Write down the expression for the length of belt for open belt drive.

Ans: Length of belt for open belt drive is

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x}$$

Question No.-05:

Write down the expression for the length of belt for cross or twist belt drive.

Ans: Length of belt for cross or twist belt drive is

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

Question No.-06:

State the ratio between the belt tensions of flat belt drive.

Ans: The ratio between the belt tensions of flat belt drive is

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

Question No.-07:

What do you understand by centrifugal tension?

Ans: Since the belt is continuously runs over the pulleys, therefore, some centrifugal force is caused, whose effect is to increase the tension on both, tight as well as slack sides. The tension caused by centrifugal force is called *Centrifugal Tension*.

Question No.-08:

What is initial tension?

Ans: When the belt is wound round the two pulleys, its two ends are joined together; so that the belt may continuously move over the pulleys, since the motion of the belt from the driver and the follower is governed by a firm grip, due to friction between the belt and the pulleys. In order to increase this grip, the belt is tightened up. At this stage, even when the pulleys are stationary, the belt is subjected to some tensions called initial tension.

Question No.-09:

What is crowning of pulley?

Ans: A crowned pulley is a pulley that has a slight hump in the middle, tapering off ever so slightly towards either edge.

A crowned pulley is used to increase the stability of the belt on the pulley and to run at the center of the pulley without slipping off from the edges.

Question No.-10:

What is circular pitch?

Ans: It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by $\boldsymbol{p_c}$.

Mathematically,

$$p_c = \frac{\pi D}{T}$$

Question No.-11:

Define diametral pitch.

Ans: It is the ratio of number of teeth to the pitch circle diameter in millimetres. It is denoted by $\boldsymbol{p_d}$.

Mathematically,

$$p_d = \frac{T}{D} = \frac{\pi}{p_c}$$

Question No.-12:

Define module.

Ans: It is the ratio of the pitch circle diameter in millimeters to the number of teeth. It is usually denoted by $\boldsymbol{m}$.

Mathematically,

$$m = \frac{D}{T} = \frac{1}{p_d}$$

Question No.-13:

Define gear train and classify it.

Ans: The combination of gear wheels or gears, used to increase or decrease the speed of the driven shaft is called gear train.

Gear trains are classified as follows:

1. Simple gear train
2. Compound gear train
3. Reverted gear train
4. Epicyclic gear train

Long Type Questions

1. Derive an expression for length of belt for open belt drive.
2. Derive an expression for length of belt for cross belt drive.
3. Derive an expression for ratio of belt tensions.
4. Write short notes on v- belt and v-belt drives.
5. Describe the working principle of epicyclic gear train.
6. Describe the working principle of reverted gear train.
7. All numerical.

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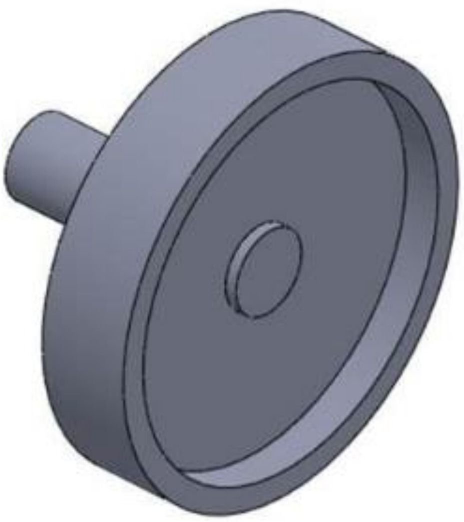
CHAPTER NO.- 04

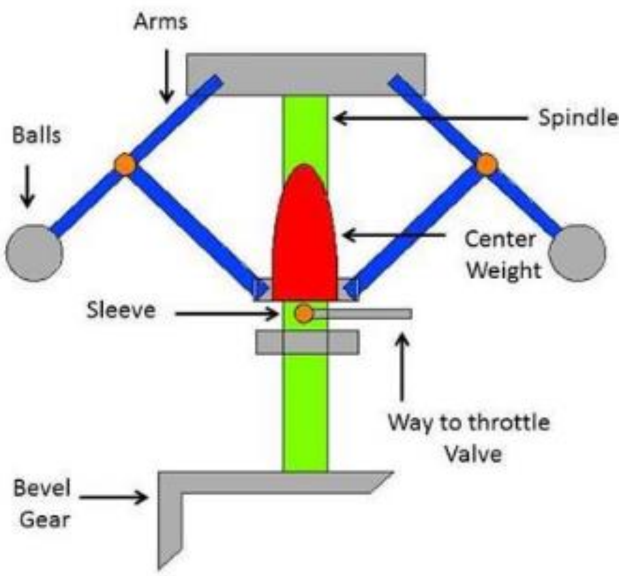
GOVERNORS & FLYWHEELS

Article No.	Name of the Article
4.1	Function of governor
4.2	Classification of governor
4.3	Working of Watt governors.
	Working of Porter governors.
	Working of Proell governors.
	Working of Hartnell governors
4.4	Conceptual explanation of sensitivity, stability and isochronism.
4.5	Function of flywheel.
4.6	Comparison between flywheel & governor.
4.7	Fluctuation of energy and coefficient of fluctuation of speed.
	Numerical problem



Flywheel

VS



Governor

CHAPTER 3.0

GOVERNORS & FLYWHEELS

4.1: Function of Governor:

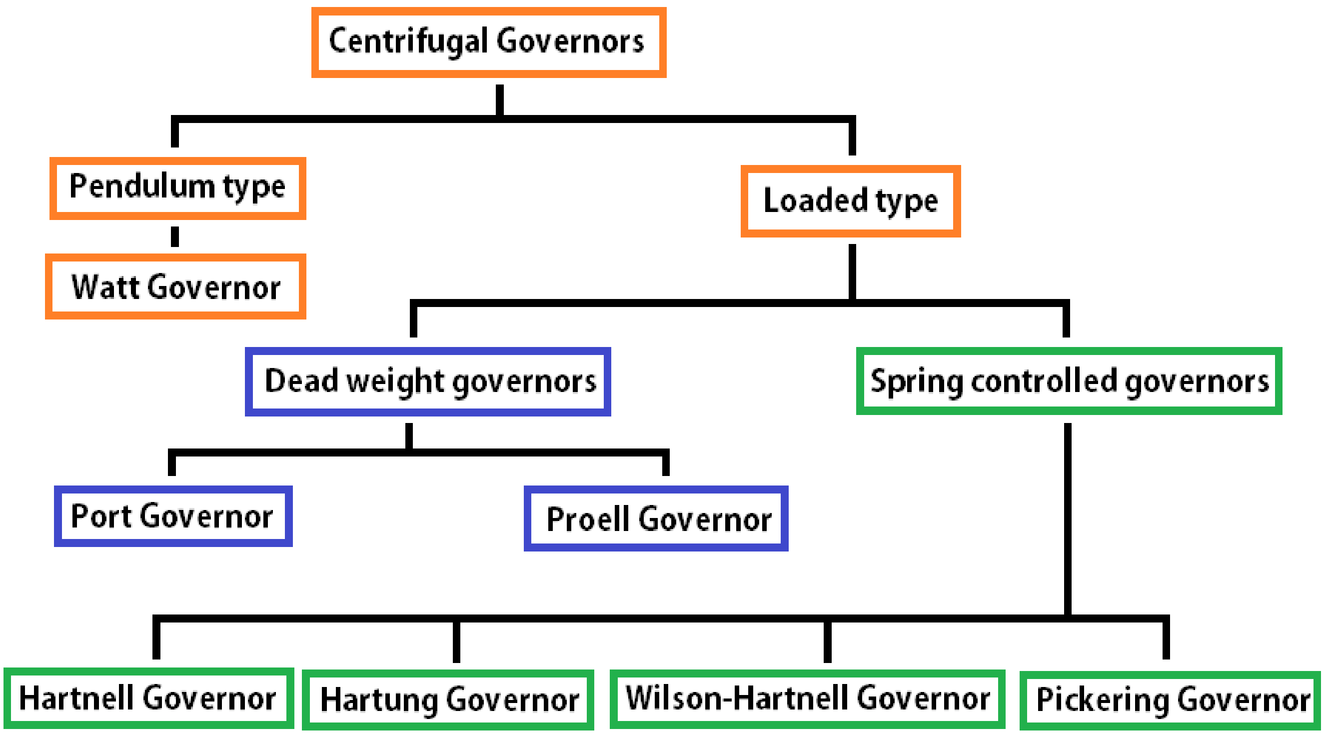
The function of a governor is to regulate the mean speed of an engine, when there are variations in the load e.g., when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required. The governor automatically controls the supply of working fluid to the engine with the varying load conditions and keeps the mean speed within certain limits.

4.2: Classification of Governor:

The governors may, broadly, be classified as:

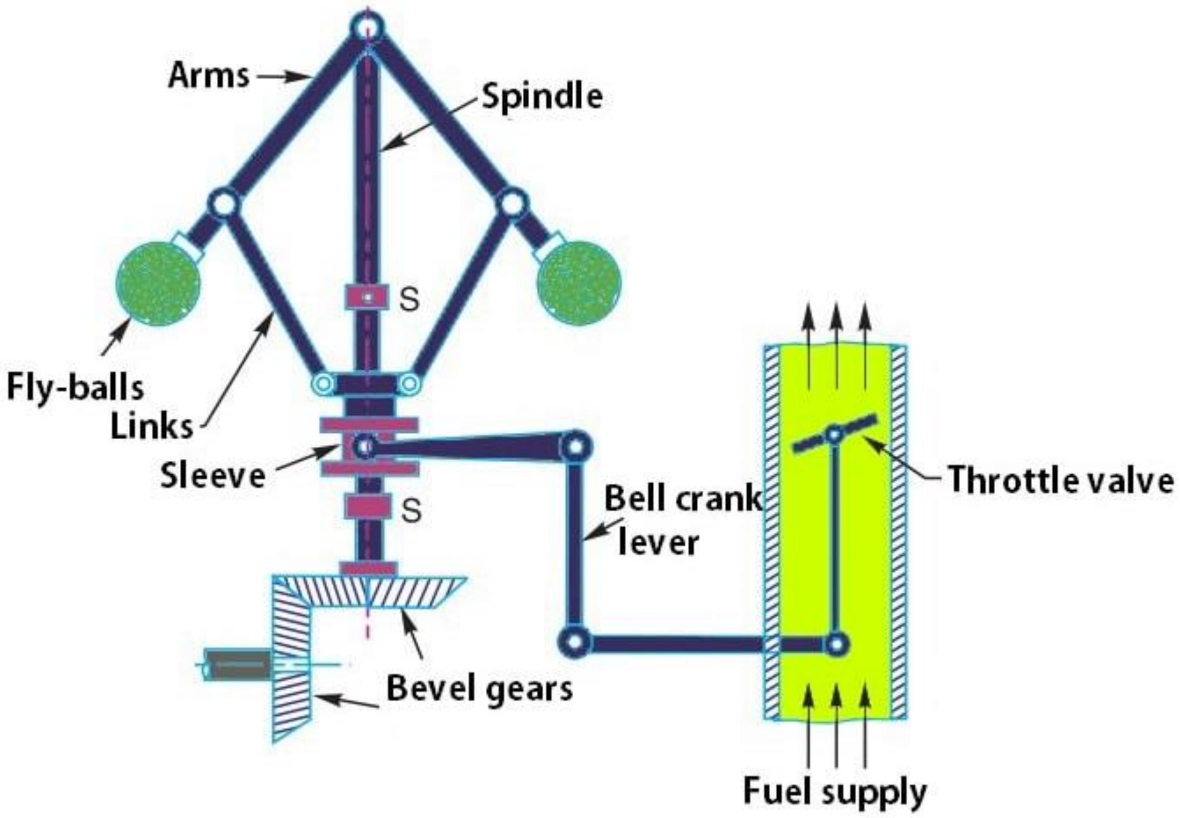
1. Centrifugal governors
2. Inertia governors

The centrifugal governors, may further be classified as follows:



4.3: Working of Watt Governors:

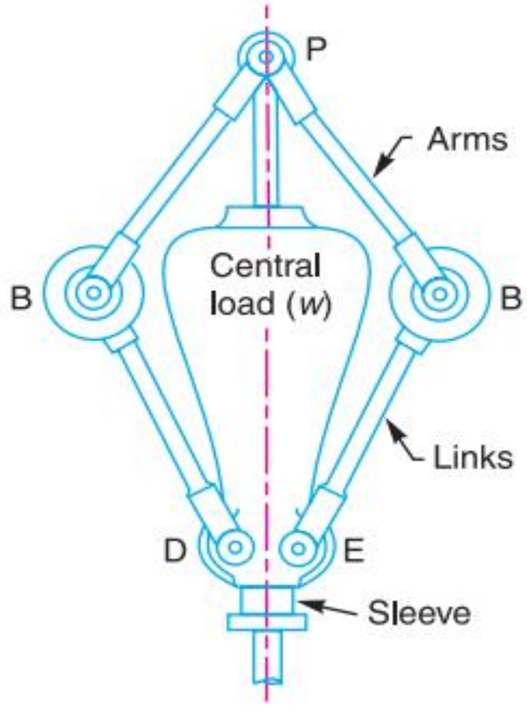
The simplest form of a centrifugal governor is a Watt governor. The centrifugal governors are based on the balancing of centrifugal force on the rotating balls by an equal and opposite radial force, known as the controlling force (i.e., the force which is provided either by the action of gravity as in Watt/ Porter/ Proell governor or by a spring as in case of Hartnell governor).



It consists of two balls of equal mass, which are attached to the ***arms***. These balls are known as ***governor balls*** or ***fly balls***. The balls revolve with a ***spindle***, which is driven by the engine through ***bevel gears***. The upper ends of the arms are pivoted to the spindle, so that the balls may rise up or fall down as they revolve about the vertical axis. The arms are connected by the ***links*** to a ***sleeve***, which is keyed to the spindle.

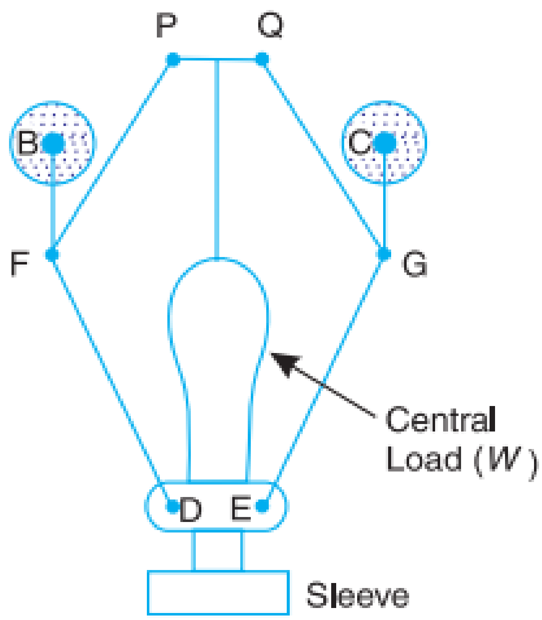
This sleeve revolves with the spindle; but can slide up and down. The balls and the sleeve rise when the spindle speed increases, and falls when the speed decreases. In order to limit the travel of the sleeve in upward and downward directions, two stops **S** & **S** are provided on the spindle. The sleeve is connected by a ***bell crank lever*** to a ***throttle valve***. The supply of the working fluid decreases when the sleeve rises and increases when it falls.

Working of Porter Governors:



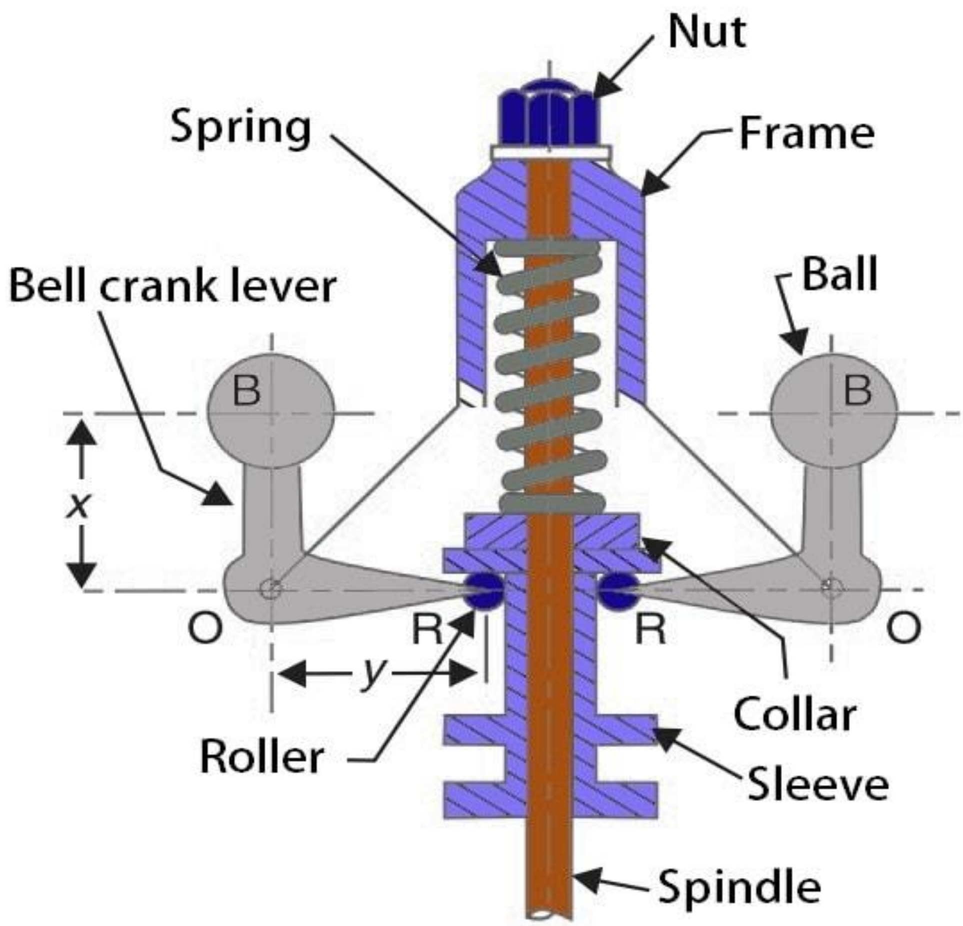
The Porter governor is a modification of a Watt's governor, with central load attached to the sleeve. The load moves up and down the central spindle. This additional downward force increases the speed of revolution required to enable the balls to rise to any predetermined level.

Working of Proell Governors:



It is a modification of Porter's Governor. The Proell governor has the balls fixed at B and C to the extension of the links DF and EG. The arms FP and GQ are pivoted at P and Q respectively.

Working of Hartnell Governors:



A Hartnell governor is a spring-loaded governor. It consists of ***two bell crank levers*** pivoted at the points **O** & **O** to the frame. The frame is attached to the governor spindle and therefore rotates with it. Each lever carries a ball at the end of the vertical ***arm OB*** and a roller at the end of the horizontal ***arm OR***. A helical spring in compression provides equal downward forces on the ***two rollers (R & R)*** through a ***collar*** on the sleeve. The spring force may be adjusted by screwing a nut up or down on the sleeve.

4.4: Sensitivity, Stability & Isochronism:

Sensitivity of Governors:

Consider two governors A and B running at the same speed. When this speed increases or decreases by a certain amount, the lift of the sleeve of governor A is greater than the lift of the sleeve of governor B. It is then said that the governor A is more sensitive than the governor B.

Sensitivity may be defined as the ratio of the difference between the maximum and minimum equilibrium speeds to the mean equilibrium speed.

Let

N_1 = Minimum equilibrium speed,

N_2 = Maximum equilibrium speed,

N = Mean equilibrium speed = $\frac{N_1 + N_2}{2}$

∴ Sensitivity of governors

$$= \frac{N_2 - N_1}{N} = \frac{2\,(N_2 - N_1)}{N_1 + N_2} = \frac{2\,(\omega_2 - \omega_1)}{\omega_1 + \omega_2}$$

Stability of Governors:

A governor is said to be stable when for every speed within the working range there is a definite configuration i.e., there is only one radius of rotation of the governor balls at which the governor is in equilibrium. For a stable governor, if the equilibrium speed increases, the radius of governor balls must also increase.

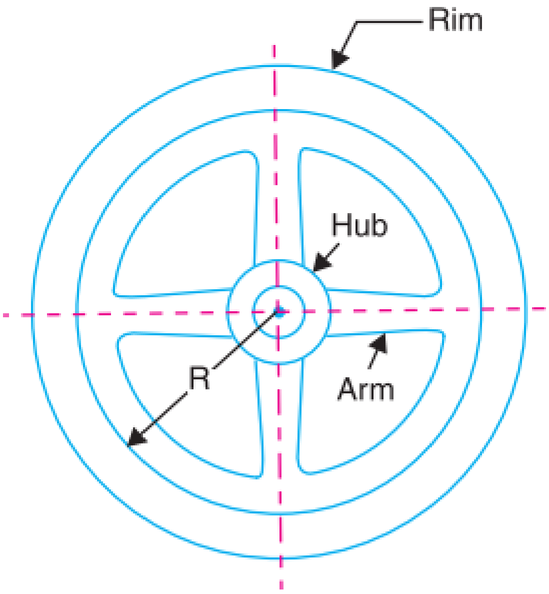
A governor is said to be unstable, if the radius of rotation decreases as the speed increases.

Isochronism of Governors:

A governor is said to be isochronous when the equilibrium speed is constant (i.e., range of speed is zero) for all radii of rotation of the balls within the working range, neglecting friction. The isochronism is the stage of infinite sensitivity.

4.5: Function of Flywheel:

A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.



In other words, a flywheel controls the speed variations caused by the fluctuation of the engine turning moment during each cycle of operation.

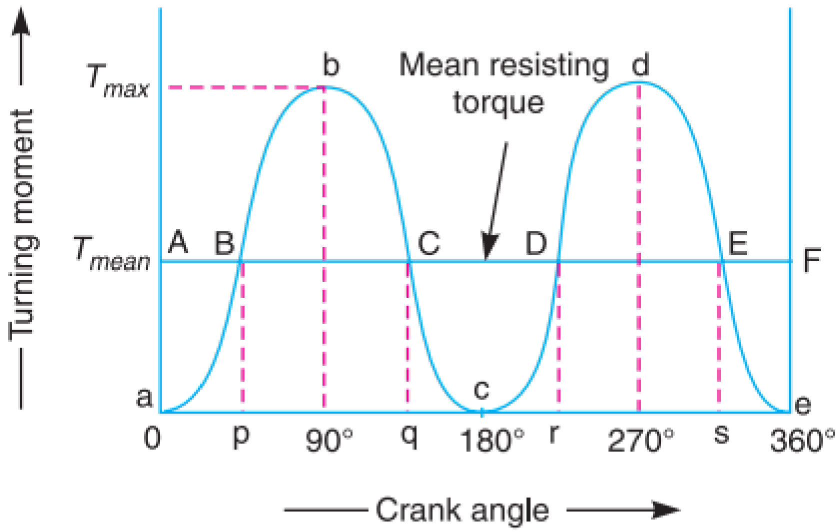
4.6: Comparison between Flywheel & Governor:

Flywheel	Governor
It stores the energy and gives up the energy whenever required during cycle.	It regulates the speed by regulating the quantity of fuel supplied to the engine.
It has no control over the quantity of working fluid.	It controls the quantity of working fluid.
It regulates the speed during one cycle only.	It regulates the speed over period of time.
It is not an essential element for every prime mover.	It is an essential element for every prime mover.
It is used in toys, IC engines, etc.	It is used in automobile vehicles, etc.

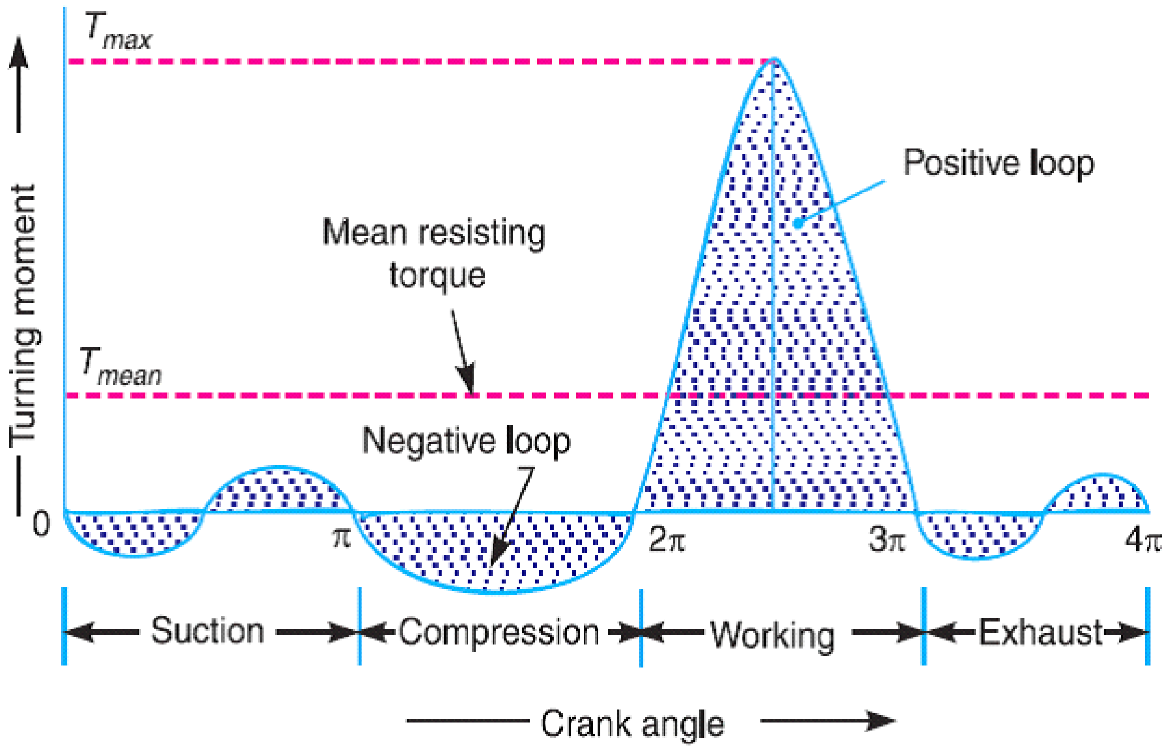
4.7: Fluctuation of energy and coefficient of fluctuation of speed.

Fluctuation of energy:

The fluctuation of energy may be determined by the turning moment diagram for one complete cycle of operation. Consider the turning moment diagram for a single cylinder double acting steam engine as shown in Figure below.



The variations of energy above and below the mean resisting torque line are called fluctuations of energy. The areas BbC, CcD, DdE, etc. represent fluctuations of energy.



A little consideration will show that the engine has a maximum speed either at q or at s. This is due to the fact that the flywheel absorbs energy while the crank moves from **p** to **q** and from **r** to **s**. On the other hand, the engine has a minimum speed either at **p** or at **r**. The reason is that the flywheel gives out some of its energy when the crank moves from **a** to **p** and **q** to **r**. The difference between the maximum and the minimum energies is known as ***maximum fluctuation of energy***.

Coefficient of fluctuation of speed:

The difference between the ***maximum and minimum speeds*** during a cycle is called the ***maximum fluctuation of speed***. The ratio of the maximum fluctuation of speed to the mean speed is called the ***coefficient of fluctuation of speed.***

Little Further:

1. **Fluctuation of energy,**

$$\Delta E = \frac{\pi^2}{900} \times mk^2.N(N_1 - N_2) = \frac{\pi^2}{900} \times mk^2.N^2 C_s$$

2. **Coefficient of fluctuation of speed,**

$$C_s = \frac{N_1 - N_2}{N}$$

3. **Mean speed during the cycle**

$$N = \frac{N_1 + N_2}{2}$$

4. **Moment of Inertia of flywheel**

$$I = mk^2$$

Numerical No.- 01

The mass of flywheel of an engine is 6.5 tones and the radius of gyration is 1.8 meters. It is found from the turning moment diagram that the fluctuation of energy is 56 kN-m. If the mean speed of the engine is 120 r.p.m., find the maximum and minimum speeds.

Solution:

Given:

m = 6.5 t = 6500 Kg ; k = 1.8 m ; ΔE = 56 kN-m = 56 x 10^3 N-m ; N = 120 rpm

Let N_1 and N_2 = Maximum and minimum speeds respectively.

We know that fluctuation of energy (Δ E),

$$56 \times 10^3 = \frac{\pi^2}{900} \times mk^2 . N(N_1 - N_2)$$

$$= \frac{\pi^2}{900} \times 6500 \times 1.8^2 \times 120(N_1 - N_2) = 27{,}715(N_1 - N_2)$$

$$\therefore N_1 - N_2 = \frac{56 \times 10^3}{27{,}715} = 2\, rpm$$

We know that, Mean speed (N)

$$120 = \frac{N_1 + N_2}{2} \; or \; N_1 + N_2 = 120 \times 2 = 240\, rpm$$

From the above equations, we get

$$N_1 = 121\, rpm \;\&\; N_2 = 119\, rpm \; (Ans)$$

Short Type Questions with Answer

Question No.-01:

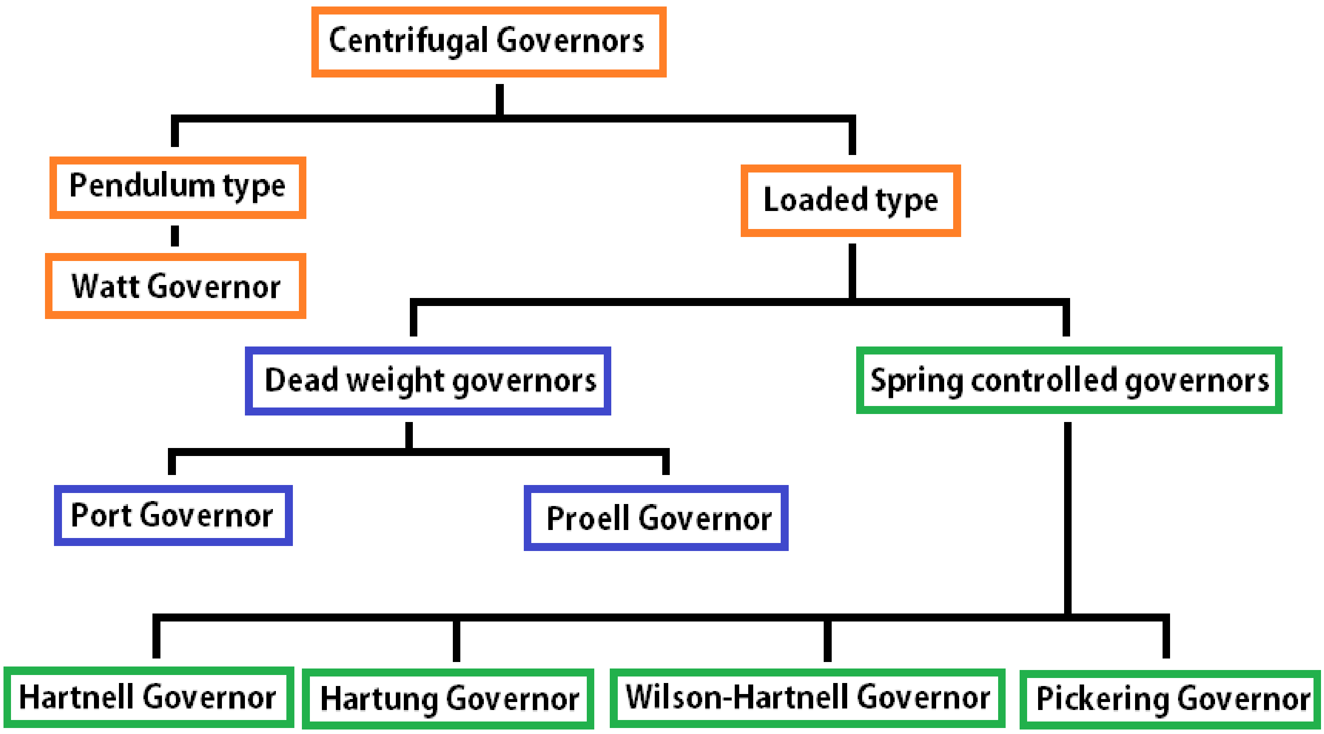
State the function a governor.

Ans: The function of a governor is to regulate the mean speed of an engine, when there are variations in the load e.g., when the load on an engine increases, its speed decreases, therefore it becomes necessary to increase the supply of working fluid. On the other hand, when the load on the engine decreases, its speed increases and thus less working fluid is required.

Question No.-02:

Classify centrifugal governor.

Ans:



Question No.-03:

State the function a flywheel.

Ans: A flywheel used in machines serves as a reservoir, which stores energy during the period when the supply of energy is more than the requirement, and releases it during the period when the requirement of energy is more than the supply.

Question No.-04:

Differentiate between Flywheel and Governor.

Ans:

Flywheel	Governor
It stores the energy and gives up the energy whenever required during cycle.	It regulates the speed by regulating the quantity of fuel supplied to the engine.
It has no control over the quantity of working fluid.	It controls the quantity of working fluid.
It regulates the speed during one cycle only.	It regulates the speed over period of time.
It is not an essential element for every prime mover.	It is an essential element for every prime mover.
It is used in toys, IC engines, etc.	It is used in automobile vehicles, etc.

Question No.-05:

Define co-efficient of fluctuation of speed.

Ans: The difference between the ***maximum and minimum speeds*** during a cycle is called the ***maximum fluctuation of speed***. The ratio of the maximum fluctuation of speed to the mean speed is called the ***coefficient of fluctuation of speed.***

Long Type Questions

1. Explain the working of Watt Governor with neat sketch.
2. Explain the working of Porter/ Proell Governor with neat sketch.
3. Explain the working of Hartnell Governor with neat sketch.
4. Write short note on fluctuation of energy.
5. Numerical

CHAPTER NO.- 05

BALANCING OF MACHINES

Article No.	Name of the Article
5.1	Concept of static and dynamic balancing.
5.2	Static balancing of rotating parts.
5.3	Principles of balancing of reciprocating parts.
5.4	Causes and effect of unbalance.
5.5	Difference between static and dynamic balancing.

CHAPTER 5.0

BALANCING OF MACHINES

5.1: Concept of Static and Dynamic Balancing:

Introduction to Balancing:

Now a day, there is an increasing trend towards manufacturing of high-speed machinery of all kind such as, engines and CNC machines, etc so as to complete the same work in a lesser time. It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible. If these parts are not properly balanced, the dynamic forces are set up. These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations in them.

Study of balancing involves two types of problems, namely:

1. Balancing of rotating masses
2. Balancing of reciprocating masses

Static Balancing:

The static or standing balance of rotating masses exist if there are in equilibrium among themselves, when not running, regardless of the position in which the masses may be placed.

In other words, while in static balance, the centre of gravity of all the rotating masses remain in a fixed position relative to the frame of the machine when not running, regardless of the position of such masses.

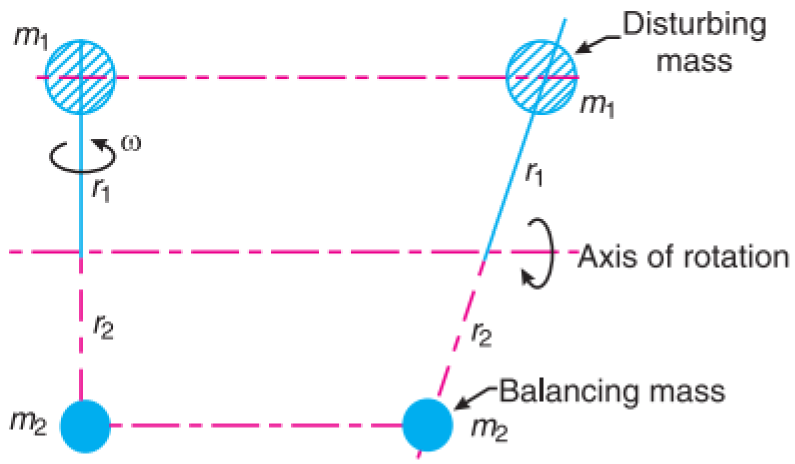
Dynamic Balancing:

The dynamic balance exists when inertia forces and couples exerted by the rotating masses are in equilibrium among themselves.

5.2: Static Balancing of Rotating Parts:

Balancing of Rotating masses or parts:

The process of providing counteracting mass/ masses to nullify the effect of unbalanced centrifugal force or forces caused by a rotating mass / masses, is called ***Balancing of Rotating Masses***.



When several masses rotate in a single plane and if the resultant of all the centrifugal forces is zero, the system is said to be statically balanced. The process followed to achive this is known as ***static balancing***.

In static balancing, mathematically,

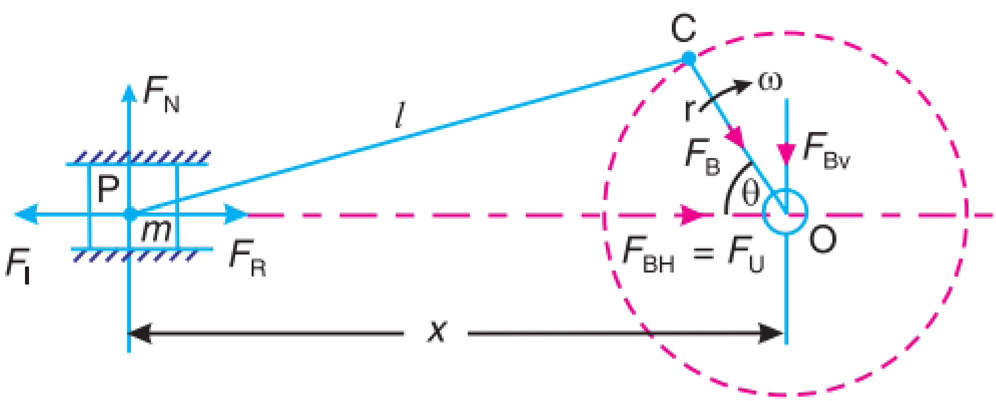
$$\sum F_c = 0$$

Which means that algebric sum of all the centrifugal forces of the system is zero.

5.3: Principles of balancing Reciprocating Parts:

We know that, the various forces acting on the reciprocating parts of an engine. The resultant of all the forces acting on the body of the engine due to inertia forces only is known as unbalanced force or shaking force. Thus, if the resultant of all the forces due to inertia effects is zero, then there will be no unbalanced force, but even then an unbalanced couple or shaking couple will be present.

Consider a horizontal reciprocating engine mechanism as shown in Figure below.



Let

F_R = Force required to accelerate the reciprocating parts,

F_I = Inertia force due to reciprocating parts,

F_N = Force on the sides of the cylinder walls or normal force acting on the cross-head guides,

F_B = Force acting on the crankshaft bearing or main bearing.

Since F_R and F_I are equal in magnitude but opposite in direction, therefore they balance each other. The horizontal component of F_B (i.e. F_{BH}) acting along the line of reciprocation is also equal and opposite to F_I . This force $F_{BH} = F_U$ is an unbalanced force or shaking force and required to be properly balanced.

The force on the sides of the cylinder walls (F_N) and the vertical component of F_B (i.e., F_{BV}) are equal and opposite and thus form a shaking couple of magnitude $F_N \times x$ or $F_{BV} \times x$.

From above we see that the effect of the reciprocating parts is to produce a shaking force and a shaking couple. Since the shaking force and a shaking couple vary in magnitude and direction during the engine cycle, therefore they cause very objectionable vibrations.

Thus, the purpose of balancing the reciprocating masses is to eliminate the shaking force and a shaking couple. In most of the mechanisms, we can reduce the shaking force and a shaking couple by adding appropriate balancing mass, but it is usually not practical to eliminate them completely. In other words, the reciprocating masses are only partially balanced.

5.4: Causes and Effects of Unbalance:

Causes:

1. Loss of mass from the machine parts.
2. The shape of rotating shaft is unsymmetrical.
3. The material is not uniform, especially in casting.
4. Any deformation is existing due to distortion, etc.
5. Improper assembly.
6. Addition of key or key ways.

Effects:

1. Major effect of unpleasant and dangerous vibration in machines.
2. Noise produced.
3. Increases the load on bearings.
4. Setting up of undesired stress in machine parts.
5. Increased maintenance.
6. Decreased machine life.

5.5: Difference between static and Dynamic Balancing:

Static Balance	Dynamic Balance
This is the ability of a system to balance while at rest.	This is the ability of a system to balance while in motion.
The net dynamic force acting on the shaft is equal to zero.	The net couple due to the dynamic forces is equal to zero.
In this case, the center of the masses of the system must lie on the axis of rotation.	In this case, algebraic sum of the moments about any point in the plane must be zero.

************************ END *************************

Short Type Questions with Answer

Question No.-01:

Define static balancing.

Ans: It may be defined as the process to balance several masses rotating in a single plane when the resultant of all the centrifugal forces is zero.

Question No.-02:

Define dynamic balancing.

Ans: It may be defined as the process to balance several rotating masses lying in different parallel planes, when the resultant of centrifugal forces and couples is zero.

Question No.-03:

State the causes of unbalancing.

Ans: The causes of unbalance in machine parts are:

1. Loss of mass from the machine parts.
2. The shape of rotating shaft is unsymmetrical.
3. The material is not uniform, especially in casting.
4. Any deformation is existing due to distortion, etc.
5. Improper assembly.
6. Addition of key or key ways.

Long Type Questions

1. Differentiate between static and dynamic balancing.
2. Write short note on causes and effect of unbalance.
3. Explain about the need of balancing of reciprocating masses or parts.

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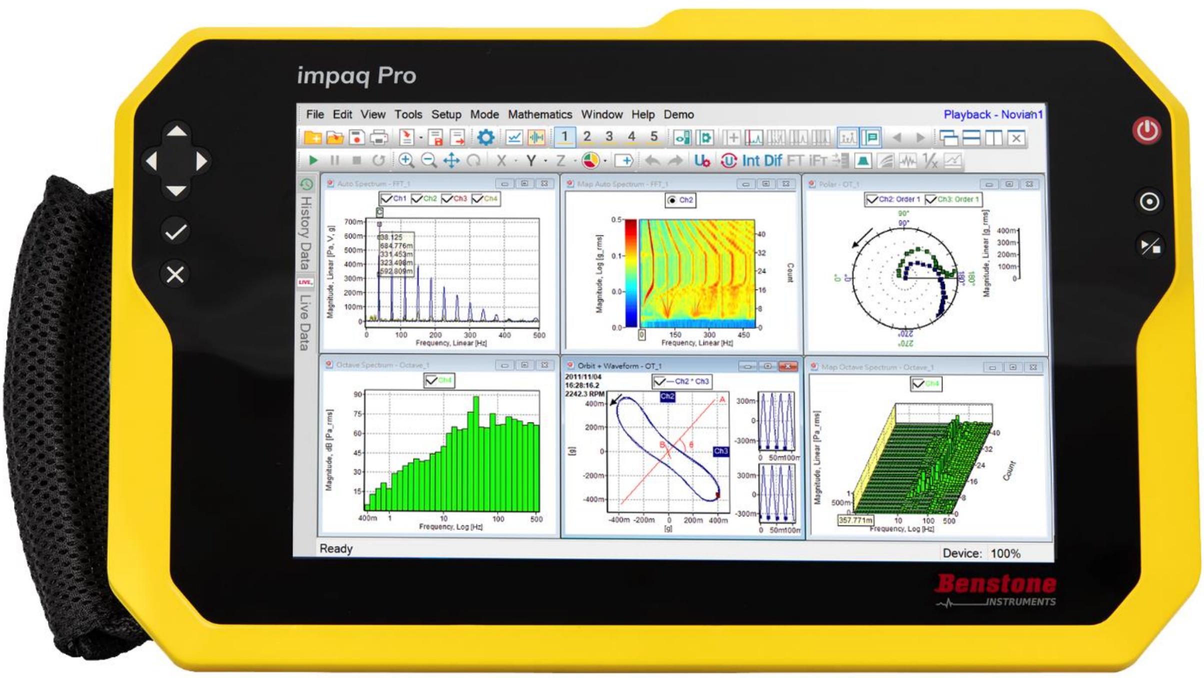
CHAPTER NO.- 06

VIBRATION OF MACHINE PARTS

Article No.	Name of the Article
6.1	Introduction to vibration and related terms (Amplitude, Time Period, Cycle & Frequency)
6.2	Classification of Vibration
6.3	Basic concept of Natural, Damped & Forced Vibration
6.4	Torsional and Longitudinal Vibration
6.5	Causes and Remedies of Vibration



(Vibration Analyser)

CHAPTER 6.0

VIBRATION OF MACHINE PARTS

6.1: Introduction to Vibration:

When elastic bodies such as a spring, a beam and a shaft are displaced from the equilibrium position by the application of external forces, and then released, they execute a ***vibratory motion*** or ***vibration***.

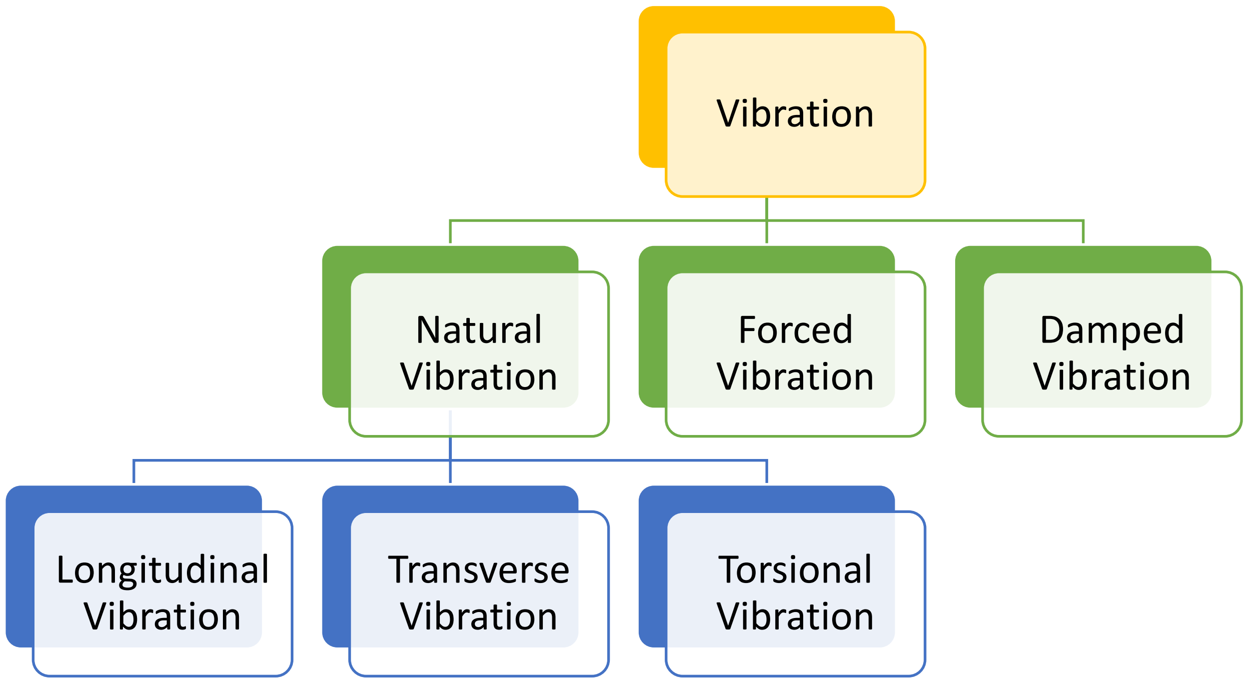
This is due to the reason that, when a body is displaced, the internal forces in the form of elastic or strain energy are present in the body. At release, these forces bring the body to its original position. When the body reaches the equilibrium position, the whole of the elastic or strain energy is converted into kinetic energy due to which the body continues to move in the opposite direction. The whole of the kinetic energy is again converted into strain energy due to which the body again returns to the equilibrium position. In this way, the vibratory motion is repeated indefinitely.

6.1: Important terms connected with Vibration:

The following terms are commonly used in connection with the vibration:

1. **Period of vibration or Time Period:**
   It is the time interval after which the motion is repeated itself. The period of vibration is usually expressed in seconds.
2. **Cycle:**
   It is the motion completed during one time period.
3. **Frequency:**
   It is the number of cycles described in one second. In S.I. units, the frequency is expressed in hertz (briefly written as Hz) which is equal to one cycle per second.
4. **Amplitude:**
   The maximum displacement or distance moved by a point on a **vibrating** body or wave measured from its equilibrium position. It is usually expressed in meters.

6.2: Classification of Vibration:



6.3: Basic Concept Natural Vibration, Forced Vibration & Damped Vibration:

1. Natural Vibration or Free Vibration:

When no external force acts on the body, after giving it an initial displacement, then the body is said to be under ***free or natural vibrations***. The frequency of the free vibrations is called ***free or natural frequency***.

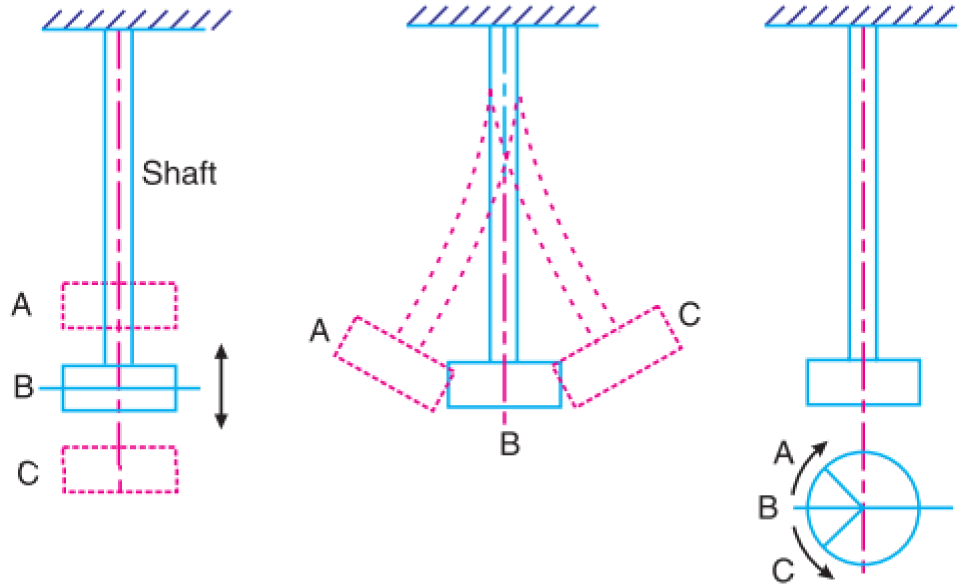
2. Forced Vibration:

When the body vibrates under the influence of external force, then the body is said to be under ***forced vibrations***. The external force applied to the body is a periodic disturbing force created by ***unbalance***. The vibrations have the same frequency as the applied force.

3. Damped Vibration:

When there is a reduction in amplitude over every cycle of vibration, the motion is said to be ***damped vibration***. This is due to the fact that a certain amount of energy possessed by the vibrating system is always dissipated in overcoming frictional resistances to the motion.

6.4: Longitudinal, Transverse & Torsional Vibration:



B = Mean position ; A and C = Extreme positions.

(*a*) Longitudinal vibrations. (*b*) Transverse vibrations. (*c*) Torsional vibrations.

1. **Longitudinal Vibration:**

   When the particles of the shaft or disc moves parallel to the axis of the shaft, as shown in Figure (a), then the vibrations are known as ***longitudinal vibrations***. In this case, the shaft is elongated and shortened alternately and thus the tensile and compressive stresses are induced alternately in the shaft.

2. **Transverse Vibration:**

   When the particles of the shaft or disc move approximately perpendicular to the axis of the shaft, as shown in Figure (b), then the vibrations are known as ***transverse vibrations***. In this case, the shaft is straight and bent alternately and bending stresses are induced in the shaft.

3. **Torsional Vibration:**

   When the particles of the shaft or disc move in a circle about the axis of the shaft, as shown in Figure (c), then the vibrations are known as ***torsional vibrations***. In this case, the shaft is twisted and untwisted alternately and the torsional shear stresses are induced in the shaft.

6.5: Causes & Remedies of Vibration:

CAUSES	REMEDIES
• Unbalanced rotating/ reciprocating parts. • Loose fastenings of the moving parts. • Incorrect allignment (in coupling). • Worn out/ broken teeth of gears. • Loose transmission belt.	• Proper balancing the rotating/ reciprocating masses. • Proper tightning and locking of the parts. • Correcting the misallignment. • Replacing the gear. • Tightening/ Replacing the belt.

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Short Type Questions with Answer

Question No.-01:

Define vibration.

Ans: When elastic bodies such as a spring, a beam and a shaft are displaced from the equilibrium position by the application of external forces, and then released, they execute a ***vibratory motion*** or ***vibration***.

Question No.-02:

Define free vibration.

Ans: When no external force acts on the body, after giving it an initial displacement, then the body is said to be under ***free or natural vibrations***.

Question No.-03:

Define damped vibration.

Ans: When the body vibrates under the influence of external force, then the body is said to be under ***forced vibrations***. The external force applied to the body is a periodic disturbing force created by ***unbalance***.

Question No.-04:

Define longitudinal vibration.

Ans: When the particles of the shaft or disc moves parallel to the axis of the shaft, then the vibrations are known as ***longitudinal vibrations***. In this case, the shaft is elongated and shortened alternately and thus the tensile and compressive stresses are induced alternately in the shaft.

Long Type Questions

1. Differentiate between free and forced vibration.
2. Write short note on causes and remedies of vibration.
3. Explain different types of vibration with neat sketch.

THE END